Electric Currents Through Ion Tracks in Silicon Devices

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Abstract—**A modified form of Ohm's law, describing electric currents through ion tracks, is presented as a tool for future theoretical modeling efforts related to charge collection from ion tracks in silicon devices. The equation is rigorously derived from the drift/diffusion equations and accounts for all currents (electron and hole, drift, and diffusion). While only one quantitative result is given, a fairly complete description of charge collection from ion tracks in silicon diodes is qualitatively discussed.**

Index Terms—**Charge collection from ion tracks, charge transport in doped silicon, funneling, single-event effect.**

I. Introduction

SINGLE-EVENT effects can occur in a semiconductor device when an energetic charged particle moving through the device creates a column, or track, of mobile electron-hole pairs. These carriers can then produce currents, via drift/diffusion (DD), which may lead to an unwanted device response. The simplest and most familiar illustration of currents produced by ion tracks consists of a reverse-biased silicon diode containing a p–n junction depletion region (DR) above a uniformly doped quasi-neutral substrate, as shown in Fig. 1. Carriers liberated by an ion track produce a current through the diode, which can sometimes create unwanted effects in external circuits that respond to the diode current. Theoretical predictions of single-event effects require an understanding of charge transport in the presence of an ion track, and the subject of track conductivity sometimes becomes relevant. A number of investigators in the single-event effects community believe that high density ion tracks in quasi-neutral regions are nonconductive, except near the outer regions of the track where the carrier density is less than the doping density. An objective of this paper is to first argue that this is not true and then to derive a modified form of Ohm's law governing electric currents through ion tracks.

Underlying the idea that a track is relatively nonconductive is the idea that a current requires a charge separation. If this were true, an ion track in a quasi-neutral medium *would* be almost nonconductive, because quasi-neutrality severely restricts charge separation. In reality, a current does not require a charge separation because carriers leaving a volume element can be replaced by others moving in. An exception occurs when there is cylindrical symmetry with no longitudinal current. In this case there is no way to replace carriers near the track center line, so a net radial current does imply a charge separation; hence, there is no net radial current (electrons and holes are constrained to move together). This exceptional case describes a longitudinally uniform track, expanding radially by diffusion, in a device having no electrical contacts. However, for the more relevant case of a device that does have electrical contacts so that longitudinal currents can flow, there can be a strong conduction current through the track, even through track sections that remain approximately longitudinally uniform while expanding radially by diffusion. Electrons and holes need not move together to avoid a charge separation because carriers can replace other carriers. A textbook by Feldman [1] discusses this via an analogy with a traffic jam which does not stop the traffic but does cause the cars to form a high-density cluster. Cars can leave this traffic jam and be replaced by others moving in (there can be a "car current") even if the location and length of the high-density cluster does not change at all. The carrier density along an ion track is usually described by the ambipolar diffusion equation, but this equation describes only the carrier density function and says very little about carrier flow. Substituting terminology from Feldman's traffic jam analogy, we would say that ambipolar diffusion describes the car cluster density as a function of time and spatial coordinates, but says nothing about the rate that cars move through the traffic jam. There are, of course, differences between a traffic jam and charge transport. However,

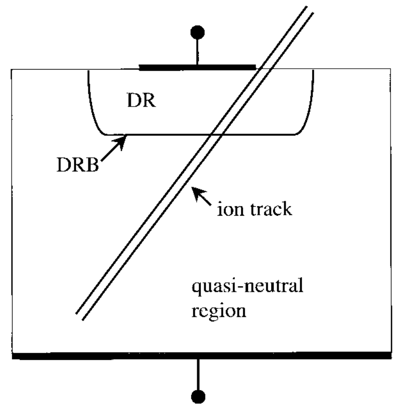

Fig. 1. A simple reverse-biased diode containing an ion track.

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whether we are discussing cars responding to traffic or carriers responding to drift/diffusion, it is still true that density and flow are different quantities, and a flow does not require a change in density. Feldman's analogy is a visual illustration of this very general statement. As previously stated, the ambipolar diffusion equation may describe the carrier density function, but says very little about carrier flow. The present paper goes a step beyond Feldman's discussion by deriving a modified form of Ohm's law, which does say something about carrier flow.

Taken by itself, the one equation derived here is not enough to calculate collected charge. Only one quantitative result is given, containing more unknowns than equations, while a complete quantitative analysis would provide as many equations as unknowns. Therefore, this result is only a tool for future theoretical work. However, this result does contribute to physical insight. Also, a fairly complete description of charge collection from ion tracks in silicon diodes is discussed qualitatively.

II. Limitations

Several sources of uncertainty should be acknowledged. The analysis to follow is based on the DD equations in a uniformly doped quasi-neutral silicon substrate. It could be argued that the DD equations may be inadequate for the extreme conditions (large carrier densities and density gradients) typical of an ion track. If true, this would have far reaching implications regarding nearly every computer simulation to date of charge collection from ion tracks. While some of the more recent computer codes utilize transport equations that are more versatile than the DD equations [2], most simulations to date are based on these equations. However, it is acknowledged that these equations could have limitations.

Another source of uncertainty is in the Einstein relation used to calculate diffusion coefficients from mobilities. The doping density will be assumed to be light enough for Maxwell–Boltzmann statistics to apply, which is one necessary condition for the classical Einstein relation to be valid, but there may still be other complications. In particular, a theoretical analysis [3] concluded that carrier–carrier scattering (CCS) affects the Einstein relation in such a way so that the ambipolar diffusion coefficient is not affected by CCS, even though mobilities are affected. However, experimental measurements [4] indicate that the ambipolar diffusion coefficient *is* affected by CCS, and the affect is consistent with the assertion that the Einstein relation is *not* affected, i.e., the classical Einstein relation applies. Hence, there may be some uncertainty here.

Another complication is that the quasi-static Poisson equation may have some limitations if transients are too fast. This equation contains a well-defined (low-frequency) dielectric constant and may not be appropriate when transients are too fast relative to the dielectric relaxation time.

It is not the intention here to resolve the above issues. These issues are mentioned only to acknowledge that the results presented here may have limitations. The DD equations, the classical Einstein relation, and the quasi-static Poisson equation are the postulated equations for the analysis to follow. The intention is to point out what these equations imply, i.e., what happens under those conditions in which the postulated equations *do* apply. It is taken for granted here that some useful concepts can be learned by investigating the implications of these equations, even if they do have some accuracy limitations.

There is one more limitation. The electron and hole mobilities may vary throughout the quasi-neutral region (e.g., due to an electric field or CCS), but a simple analysis can only be used if these mobilities are approximated in such a way so that the ratio of the electron-to-hole mobility is spatially uniform throughout the uniformly doped quasi-neutral region. This limitation may seem more acceptable if we recall that mobility models including CCS are still highly uncertain. If we can believe that useful concepts can be learned from computer simulations using uncertain mobility models, then it is also reasonable to believe that useful concepts can be learned by investigating the analytically tractable case of a spatially uniform mobility ratio in the quasi-neutral region. Note that the motivation for analytical calculations is rarely to obtain greater quantitative accuracy than a computer simulation. The motivation is typically to discover fundamental concepts, as opposed to merely observing the final result from a complex set of interactions for some specific case. When this is the objective, analytical tractability usually has a higher priority than quantitative accuracy. Note that a spatially uniform mobility ratio is not required because it gives a device some special properties, rather it is required because it makes the equations so much easier to write.

The above limitations are the only known limitations. In particular, the mathematical steps applied to the stated assumptions are equally valid whether the excess carrier density is much less than, on the order of, or much greater than the doping density. Therefore, there are no additional restrictions regarding how large or small the carrier density may be, beyond the restrictions implicitly contained in the above limitations.

III. Distinct Device Regions

Before starting an analysis applicable to quasi-neutral regions, it should first be pointed out that such regions exist (according to the DD equations), at least for the simple device illustrated in Fig. 1. One of the things learned from computer simulations is that, even in the presence of an ion track, silicon diodes having a uniformly doped substrate show a reasonably well-defined DR boundary (DRB) separating the DR (a strong space-charge region) from a quasi-neutral region. The latter region is defined by the property that the charge imbalance (measured as a density of elementary charges) is small compared to the majority carrier density. Although the DR and quasi-neutral region have been discussed by many investigators for many years, it is not yet common practice to plot computer simulation data in a way that clearly reveals these regions. Because these regions are defined by the presence of or near-absence of a space charge, they are most clearly seen by plotting electron and hole densities, together on the same graph, against a spatial coordinate. An example is shown in Fig. 2, which plots electron and hole densities against depth along the track center line for a diode containing

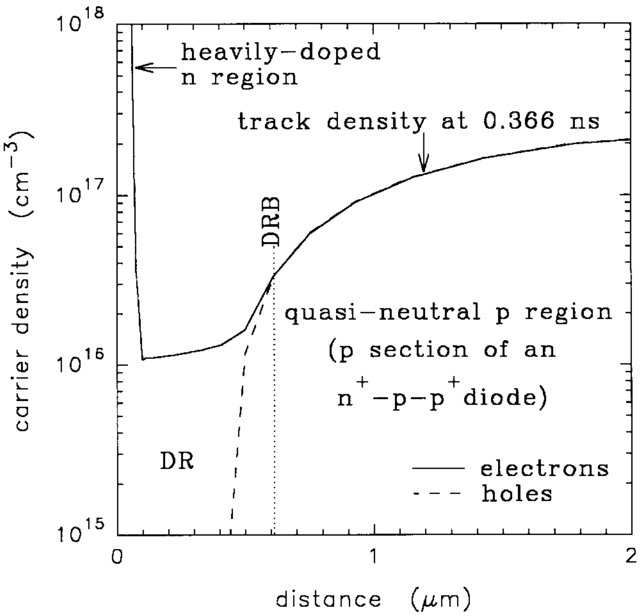

Fig. 2. A plot of carrier density illustrating a DR and quasi-neutral region separated by a DRB.

an n^+ diffusion above a p-type substrate, and with a normal incident track hitting the center of the device. Because Fig. 2 is intended only for illustration, we omit a detailed discussion of the computer simulation code and input data. Interested readers can refer to an earlier paper for this discussion [5]. The present Fig. 2 uses data from the Baseline case at $t = 0.366$ ns in the earlier paper and shows an n^+–p portion of an n^+-p-p^+ epi diode. The carrier densities greatly exceed the doping density ($< 10^{15}/\text{cm}^3$) near the DRB in this figure, so quasi-neutrality is recognized by the condition that the electron and hole densities be nearly equal. Note the striking contrast between two distinct regions in this figure, making a DRB reasonably well defined.

The DR width in the device represented by Fig. 2 was about 3 μm prior to the ion hit, while the width shown in the figure is less than 1 μm. The DR is smaller (and the quasi-neutral region larger) following an ion hit than it was prior to the hit. This is because a strong electric field in the DR quickly produces a redistribution of carriers liberated by the ion track, until many of the previously unshielded impurity ions become shielded by carriers. A portion of the device contained unshielded impurity ions prior to the hit, which then became shielded after the charge rearrangement. This portion was a space-charge region prior to the hit but becomes quasi-neutral after the rearrangement, i.e., part of the pre-ion-hit DR becomes part of the post-ion-hit quasi-neutral region. Hence the DR shrinks and the quasi-neutral region expands, so the DRB initially moves up in Fig. 1. After this initial collapse, the DRB moves down (it is moving to the right in Fig. 2) as the DR recovers and regains its initial width. Until this recovery is complete, the DR is in a partially collapsed state. The time required for a complete recovery is typically measured in nanoseconds.

It is interesting that there are virtually no majority carriers in the DR in Fig. 2, except very close to the DRB. The vertical scale could be extended many more decades downward, and the majority carrier density would still be out of the plotted range. The explanation is that after the majority carriers are driven out during the initial collapse, they are not replaced. They cannot move more than a small distance into the DR from the quasi-neutral region because of a strong opposing electric field in the DR. If the DRB were stationary, it would block the majority carrier current. There are some minority carriers in the DR because minority carriers can enter from the quasi-neutral region to replace those that have been driven out to the left in Fig. 2. However, the minority carrier density in the DR is much less than in the quasi-neutral region.

An implication, first discovered by Hsieh *et al.* [6] of a collapsed or partially collapsed DR, is that it supports much less device voltage than it did prior to the hit. Much of the voltage (applied plus built-in) formally across the DR is now across the quasi-neutral region. This is the phenomenon that has been called funneling [6].

It was tacitly assumed in the above discussion that the DR collapse is caused by a direct ion hit to the DR, liberating carriers within. If the tract misses the DR, there can still be a collapse producing a voltage across the quasi-neutral region. However, this collapse is gradual because carriers must first diffuse into the DR from outside to produce the collapse. In this case, there is no clear distinction between a collapse stage and recovery stage (the recovery stage is generally defined here to begin after the collapse is complete) because both are gradual processes and a DRB is reasonably well defined at all times. For the more violent case of a direct hit to the DR, the collapse is very fast because the strong electric field within the DR rearranges carriers liberated within very quickly until the screening effect is established and recovery begins. However, this initial charge rearrangement is not instantaneous, and some peculiar things might occur during this time. In particular, it is not yet clear whether a DRB is defined during this time. However, because the time duration of this process is so short, simulations (at least those performed by this author) have not yet found a significant amount of charge collection at the device terminals during this time. Nearly all charge collection occurs during the recovery stage when the DRB is reasonably well defined.

The conclusion from the above discussion is that, whether or not there is a direct ion hit to the DR, most (or all) charge collection occurs when there is a reasonably well-defined DRB, i.e., when the device contains distinct regions consisting of a DR and a quasi-neutral region. The distribution of the potential and current within the quasi-neutral region is the subject of the sections to follow. However, it can be pointed out now that quasi-neutrality merely means that the charge imbalance is small compared to the majority carrier density. This does not imply that the charge imbalance is too small to significantly contribute to the electric field. The potential distribution within the quasi-neutral region is profoundly affected by the charge imbalance in this region, in much the same way as space charges in the interior of a simple resistor having an inhomogeneous conductivity produce the irregular field pattern required by Ohm's law in such a device.

IV. P-TYPE VERSUS N-TYPE SUBSTRATES

The conclusion from the above section, that a moving DRB separates distinct device regions, will not be contradicted in this section, but one rather casual statement in the previous section does warrant some additional discussion. The statement was that a collapsed DR supports much less device voltage than it did prior to the ion hit. In reality, a DR can have a significantly reduced width and still retain nearly all of its previous voltage. Whether a DR does or does not give up a significant amount of its voltage to another device region is strongly dependent on whether the device substrate is n-type or p-type. A large voltage across the quasi-neutral region is much easier to obtain when the substrate is p-type.

Fundamental differences between n^+–p and p^+–n diodes were theoretically predicted for steady-state conditions [7], which can be obtained by replacing the impulsive ion-induced carrier generation with a track that continues to generate carriers at a rate that is constant in time. The steady-state analysis is unreliable for predicting that various phenomena will occur under transient conditions, but some predictions were found to apply to transient conditions, so it is interesting to consider what the steady-state analysis predicts. The analysis predicts that bulk diodes having either doping type will display qualitatively similar phenomena (the DR gives up much of its voltage to the quasi-neutral region) if a high-density track is contained within a sufficiently small distance from the DR. However, if this condition is not satisfied (e.g., if the track is either too far from the DR or too long), the analysis predicts that the DR will retain nearly all of its voltage for the n-type case, but not for the p-type case. This prediction is consistent with a computer simulation of an epi diode under transient conditions [5]. The track was longer than the epi-layer thickness, which qualifies as "too long" for a device with an n-type substrate. While a p-type case under the same conditions displayed a significantly reduced voltage across the DR, the DR for the n-type case retained nearly all of its voltage (although the DR width was significantly reduced), even though the carrier density greatly exceeded the doping density at the DRB. The steady-state and epi cases are both rather extreme in the sense that the amount of voltage lost by the DR is extremely small for the n-type substrate. A bulk diode under transient conditions is less extreme in this respect, but still shows that a DR can retain most of its voltage for the n-type substrate, as illustrated below.

To compare doping types, we first consider a simple n^+–p diode having the structure illustrated in Fig. 1. The diode contains a shallow $n+$ diffusion above a 100-μm-thick p-type substrate and is reverse biased at 5 V. The simulated track was from a normal incident center hit, is 35 μm long, and the ion linear energy transfer (LET) is 40 MeV-cm^2/mg. The computer simulation code and input data are discussed in more detail in an earlier paper [5]. The present case is identified as the "bulk version" in the earlier paper. The current peaks at about 0.6 ns after the track formation, so this may be an interesting time point at which to look at the carrier density and potential distribution. The carrier density along the track center line at this time point is plotted against depth into the device in Fig. 3.

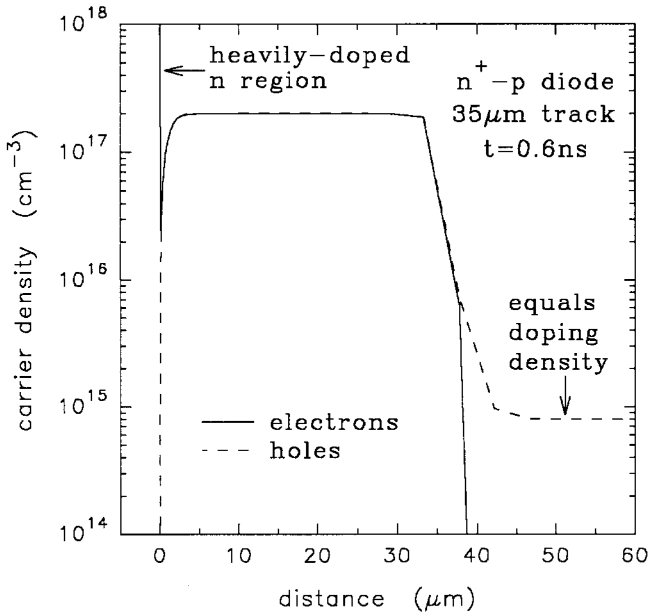

Fig. 3. A plot of carrier density for the n^+–p diode at 0.6 ns after the formation of a 35-μm track.

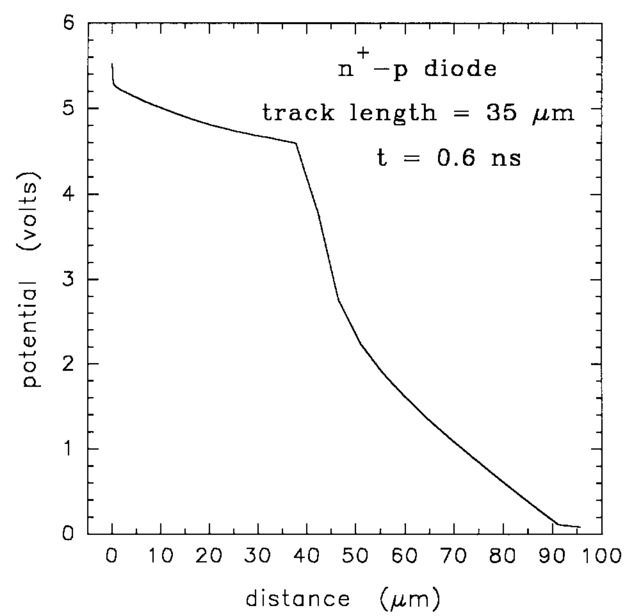

Fig. 4. A plot of potential for the n^+–p diode at 0.6 ns after the formation of a 35-μm track.

Although not visible with the resolution shown in the figure, the DR width is about 0.1 μm (readers wondering why the DR is much wider at 0.366 ns in Fig. 2 than at 0.6 ns in Fig. 3 are reminded that Fig. 2 shows a section of an epi diode, which recovers faster than a bulk diode). The potential along the track center line is plotted against depth in Fig. 4. Note that only a small voltage is across the DR, and most of the device voltage is across the section of quasi-neutral region below the track. The potential difference across the entire device interior includes the built-in potential, so this potential difference is slightly larger than the applied 5 V.

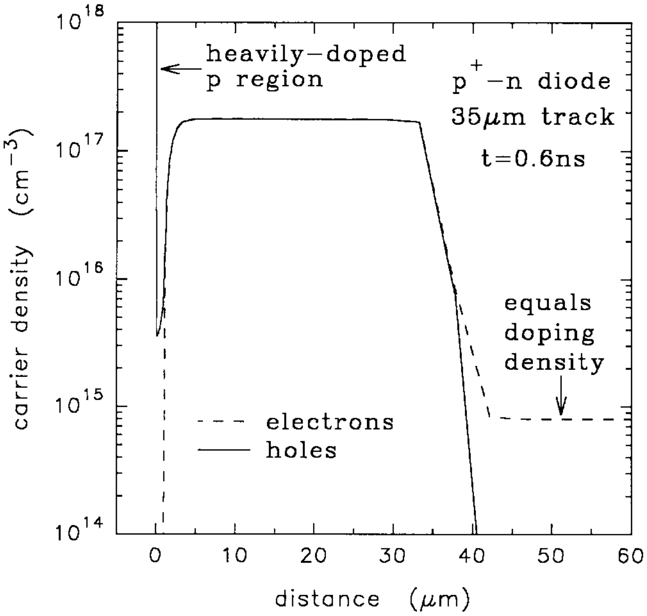

Fig. 5. A plot of carrier density for the p^+–n diode at 0.6 ns after the formation of a 35-μm track.

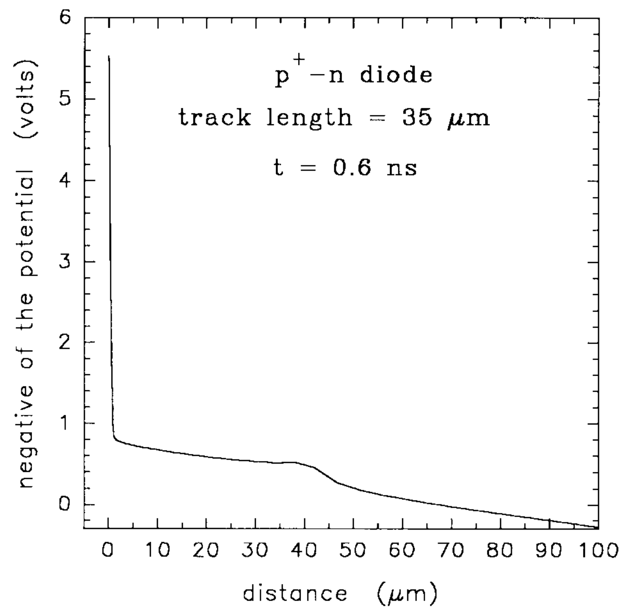

Fig. 6. A plot of potential for the p^+–n diode at 0.6 ns after the formation of a 35-μm track. The sign is reversed for a more direct comparison with the other doping type.

We next consider a p^+–n diode, which is identical to the n^+–p diode except that doping types are interchanged, and the polarity of the applied voltage is reversed. Carrier densities and potentials for this case are plotted in Figs. 5 and 6. The potential is plotted with a reversed sign for a more direct comparison with the other doping type. A profound difference between the two doping types now becomes clear. The DR retained most of its voltage for this case, even though the width is greatly reduced by the ion hit (the DR width is not visible with the resolution shown in Fig. 5, but is about 1 μm, compared to about 3 μm before the ion hit). Although the DR already has most of its voltage, it is still in the process of regaining its initial width, so the DRB is moving to the right in Fig. 5.

It is interesting that, excluding the left sections of the curves (which reflect different DR widths), the carrier densities in Figs. 3 and 5 are very similar. This is because the simulations used the same electron mobility for both doping types and the same hole mobility for both doping types. Therefore, the ambipolar diffusion coefficient is the same for both cases.

There is a very slight difference between Figs. 3 and 5 near the right ends of the track, with the n^+–p case showing a slightly more abrupt change in the minority carrier density. Even though this difference is barely perceptible with the resolution shown in the figure, the reason for it is still interesting. The explanation is that longitudinal ambipolar diffusion does not apply to the n^+–p case near the lower track end. The n^+–p case exhibits a very intense electric field (implied by Fig. 4) below this depth, while the other case exhibits a much weaker field. The intense electric field for the n^+–p case opposes the downward diffusion of minority carriers. Note that majority carriers are free to enter this region. In fact, such carriers are already there, with a density equal to the doping density as required by quasi-neutrality in the absence of minority carriers. These majority carriers are moving in response to the intense electric field (the current is at its peak at this time point), but the density remains equal to the doping density because carriers leaving a volume element are replaced by others moving in. However, the absence of minority carriers together with quasi-neutrality is ultimately responsible for both minority and majority carrier densities being what they are in the lower region, so the slight difference in track structures for the two doping types can be described as the inability of minority carriers to move into the lower region for the n^+–p case.

The two doping types would exhibit significantly different carrier densities near and below the lower track end under steady-state conditions [7], but Figs. 3 and 5 show only a very small difference for this transient problem; indicating that there has not yet been enough time for a significant downward diffusion with or without an opposing electric field. The influence of this electric field on the carrier distribution may not be important if minority carriers would not have moved far into the lower region anyway. Therefore, it may be adequate to estimate the carrier density via the ambipolar diffusion equation without any modifications to account for the strong electric field inhibiting the downward diffusion.

It was shown above that the two doping types can be profoundly different in terms of how the device voltage is divided between the DR and quasi-neutral region. If funneling is defined by the condition that there is a large voltage across the quasi-neutral region, then funneling is much easier to induce for one case than the other. Fortunately, this does not imply that the two cases require different methods of analysis for calculating currents. The analysis in Sections V and VI applies to both doping types. A low-order approximation and a correction discussed in Sections VIII and IX also applies to both doping types.

V. A Modified Ohm's Law

The well-known DD equations and Poisson's equation can be written as

$$\vec{J}_h = -qD_h\vec{\nabla}(P+p_0) - q\mu_h(P+p_0)\vec{\nabla}\varphi \tag{1a}$$
$$\vec{J}_e = qD_e\vec{\nabla}(N+n_0) - q\mu_e(N+n_0)\vec{\nabla}\varphi \tag{1b}$$
$$\vec{\nabla}\cdot\vec{J}_h = -q\frac{\partial P}{\partial t} + qg - qr \tag{2a}$$
$$\vec{\nabla}\cdot\vec{J}_e = q\frac{\partial N}{\partial t} - qg + qr \tag{2b}$$
$$\varepsilon\nabla^2\varphi = q(N-P) \tag{3}$$

where n_0 and p_0 are the equilibrium electron and hole densities respectively, N and P are the excess electron and hole densities, respectively, g and r are the generation and recombination rates, respectively (assumed the same for electrons as holes), φ is the electrostatic potential, and the other symbols have the obvious meanings. We also assume the Einstein relations

$$D_h = V_T\mu_h, \qquad D_e = V_T\mu_e \tag{4}$$

where V_T is the thermal voltage (about 0.026 volts at room temperature). The analysis is applied to a uniformly doped quasi-neutral region, so the equilibrium densities have zero gradients in (1). Also, a region is quasi-neutral when the solution to the above equations can be approximated by the solution to the equations obtained from the limiting case as ε approaches zero. In this limit, (3) is replaced by $N=P$, which cannot be used to solve for φ. However, a closed system of equations (when explicit expressions are given for r and g), that is capable of solving for both P and φ, is obtained by substituting $N=P$ into (1) and (2). Doing so while using (4) gives

$$\vec{J}_h = -q\mu_h[V_T\vec{\nabla}P + (P+p_0)\vec{\nabla}\varphi] \tag{5a}$$
$$\vec{J}_e = q\mu_e[V_T\vec{\nabla}P - (P+n_0)\vec{V}\varphi] \tag{5b}$$
$$\vec{\nabla}\cdot\vec{J}_h = -q\frac{\partial P}{\partial t} + qg - qr \tag{6a}$$
$$\vec{\nabla}\cdot\vec{J}_e = q\frac{\partial P}{\partial t} - qg + qr. \tag{6b}$$

Defining the total current by $\boldsymbol{J}_T \equiv \boldsymbol{J}_h + \boldsymbol{J}_e$, we conclude from (5) and (6) that

$$\vec{J}_T = q(\mu_e - \mu_h)V_T\vec{\nabla}P - \sigma\vec{\nabla}\varphi \tag{7}$$
$$\vec{\nabla}\cdot\vec{J}_T = 0 \tag{8}$$

where the conductivity σ is defined by

$$\sigma \equiv q[\mu_h(P+p_0) + \mu_e(P+n_0)]. \tag{9}$$

Now define A and B by

$$A \equiv \frac{\mu_e n_0 + \mu_h p_0}{\mu_e + \mu_h} \approx \begin{Bmatrix} \dfrac{\mu_h}{\mu_e+\mu_h} \times [\text{doping density}] & \text{for } p\text{-type region} \\ \dfrac{\mu_e}{\mu_e+\mu_h} \times [\text{doping density}] & \text{for } n\text{-type region} \end{Bmatrix} \tag{10}$$

$$B \equiv \frac{\mu_e - \mu_h}{\mu_e + \mu_h}. \tag{11}$$

Using (9)–(11), we can write (7) as

$$\vec{J}_T = \sigma\left[\frac{B}{P+A}V_T\vec{\nabla}P - \vec{\nabla}\varphi\right]. \tag{12}$$

The mobilities may vary throughout the quasi-neutral region, but to obtain equations that are simple to write, the mobilities are assumed to be approximated in such a way so that the ratio of the electron-to-hole mobility is spatially uniform throughout the quasi-neutral region. This results in A and B being spatially uniform, so (12) can be written as

$$\vec{J}_T = -\sigma\vec{\nabla}\psi \tag{13}$$

where the modified potential ψ is defined by

$$\psi \equiv \varphi - BV_T\,\ln\left(\frac{P+A}{A}\right). \tag{14}$$

Equation (13) can be verified by substituting the definition (14) into (13), using the chain rule to expand the gradient, and comparing the result to (12).

Note that (13) is the classical form of Ohm's law, except that it contains the modified potential ψ instead of the true electrostatic potential φ. Because of this difference, we will call (13) a modified Ohm's law.

VI. Integrated Form of the Modified Ohm's Law

For any cases of practical interest, the closed boundary surrounding a quasi-neutral region will contain at least two noninsulated, or active, boundaries in which the normal component of the total current may differ from zero. For example, Fig. 1 shows two active boundaries consisting of the DRB above and an ohmic contact below. We look for an integrated form of (13) which applies to two active boundaries and is analogous to the familiar equation $\Delta V = IR$ for resistors. The discussion below can easily be generalized to many active boundaries by replacing the analog of $\Delta V = IR$ with a matrix equation, so it is sufficiently general to consider the case of two active boundaries for illustration.

The most general case having two active boundaries is shown in Fig. 7(a). The upper active boundary of the quasi-neutral region is denoted S_2, and the values of P and φ on this boundary are denoted P_2 and V_2, respectively. Note that if P or φ are not constant on S_2, it is necessary to define P_2 and/or V_2 to be some appropriately weighted surface averages of P and φ. Similar considerations apply to S_1 at which the boundary values are denoted P_1 and V_1. The arrow in Fig. 7(a) defines the sign convention for the current I; it is positive when directed from S_2 to S_1, otherwise it is negative (or zero). The general case shown as Fig. 7(a) includes many special cases, such as: the simple diode previously discussed [Fig. 7(b)], a uniformly doped substrate between two ohmic contacts [Fig. 7(c)], and an MOS capacitor [Fig. 7(d)]. The MOS capacitor produces a space-charge region associated with accumulation, depletion, or inversion, with changes in the space-charge region producing transient currents. Another special case included in the general case, but not shown in the figures, is an epi diode similar to Fig. 7(b) except that the lower boundary S_1 is the upper boundary of a high–low

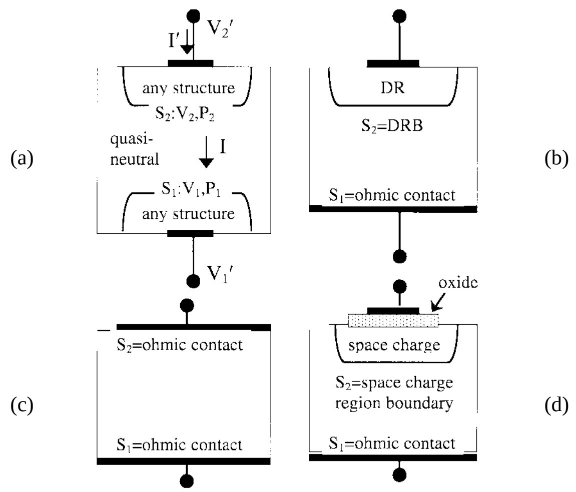

Fig. 7. (a) The most general case having two active boundaries. (b) includes many special cases, such as a diode, (c) a uniformly doped substrate between two ohmic contacts, and (d) an MOS capacitor.

junction instead of an ohmic contact. An ion track, which can be anywhere in the device, is not shown in the figures because its influence (as well as the influence from any other source of carriers) is implicitly contained in the excess carrier density function P, which determines the conductivity σ.

Note that the only difference between (13) and the classical form of Ohm's law is the symbolism; we see ψ instead of the true electrostatic potential φ. The integrated form of (13) must therefore be $\Delta\psi = IR$. Although we will end up with this equation, some justification and qualification is warranted because there is a complication. The classical integrated form of Ohm's law $\Delta V = IR$ is a dc equation. A resistor that is ideal in the sense that the point form of Ohm's law ($\boldsymbol{J} = \sigma\boldsymbol{E}$) is exact may still not be ideal in the sense that the integrated form of Ohm's law ($\Delta V = IR$) applies under transient conditions. There are two types of capacitances that may have to be included in the integrated equation under transient conditions. One type of capacitance appears whether or not σ is homogeneous. This is the capacitance between electrodes which is related to surface charge densities on the electrode surfaces. A changing surface charge on an electrode will cause the current through the medium adjacent to the electrode to differ from the current through the wire connected to the electrode, in which case it is necessary to make a distinction between I and I' in Fig. 7(a).

The other type of capacitance is relevant only if σ is inhomogeneous, but that is the case that must be considered. An inhomogeneous conductivity results in a space-charge distribution in the resistor interior in addition to surface charges at the electrodes. The influence of the space charge can be illustrated by considering a one-dimensional resistor, which simplifies the discussion. A space charge may be contained between two depths within the resistor. If this space charge changes with time, the currents at the two depths will differ. Another type of capacitance is needed to account for the space charge. Fortunately, (8) implies that we are treating a case in which this type of capacitance does not have to be included. This equation implies that the current is the same at all depths in the one-dimensional geometry (in three dimensions, the equation becomes a statement regarding surface integrals of the current, but this statement is analogous to the one-dimensional statement). There is a space charge influencing the potential distribution, but as long as the charge imbalance is small compared to the carrier density, which determines the conductivity, quasi-neutrality applies, so (8) is a valid approximation and implies that the displacement current is negligible compared to the total current.

As discussed above, (8) implies that a time-varying space charge in the quasi-neutral interior does not invalidate the classical integrated form of Ohm's law (a dc equation). However, surface charges may still have to be accounted for. Instead of accounting for the surfaces charges now by including a capacitance in the integrated equation, we can account for them later by stipulating that the current in the integrated equation be the current I in the quasi-neutral medium instead of the terminal current I' shown in Fig. 7(a). The surface charges are accounted for later when relating I to I'. The integrated equation expressed in terms of I is the classical form of Ohm's law $\Delta\psi = IR$ with R referring to the dc resistance, i.e.,

$$R \equiv \left\{\begin{array}{l}\text{The resistance, between electrodes}\\ \text{at } S_1 \text{ and } S_2\text{, describing}\\ \text{a resistor under dc conditions}\\ \text{and having the same geometry}\\ \text{as the quasi-neutral region,}\\ \text{and a conductivity equal to } \sigma\end{array}\right\}.$$

Using (14) to express $\Delta\psi$ ($\equiv \psi_2 - \psi_1$) in terms of the P's and V's, we obtain

$$V_2 - V_1 - BV_T \ln\left(\frac{P_2 + A}{P_1 + A}\right) = IR. \tag{15}$$

As previously stated, capacitances associated with boundary surfaces may require that we distinguish between I and I' in Fig. 7. The importance of this effect depends on how fast the transient is. The effect will be much less important during the gradual recovery stage than for the rapid DR collapse. In fact, analytical calculations which do not distinguish between I and I' roughly agree with simulation results [5], and the error is probably primarily from other approximations. This suggests that we may not have to distinguish between I and I' during the recovery stage. However, it usually is necessary to distinguish between ΔV ($\equiv V_2 - V_1$) and $\Delta V'$ ($\equiv V_2' - V_1'$) in Fig. 7, due to voltages across other structures in addition to contact potentials at electrode boundaries. For example, in Fig. 7(b) there will be a voltage across the DR in addition to contact potentials at the electrodes. These voltages subtract out in equilibrium, so that $\Delta V = \Delta V' = 0$, but $\Delta V \neq \Delta V'$ under nonequilibrium conditions. It is therefore important to remember that the voltages in (15) are in the quasi-neutral region at the boundaries; they are not terminal voltages (except for some special cases).

Because the voltages in (15) are usually not terminal voltages, the analysis presented here is analogous to finding an equation describing only one circuit element in a circuit containing several elements. A complete circuit cannot be analyzed until we have equations describing all circuit elements, but each analysis of another circuit element is another important tool for a future analysis of the complete circuit. Similarly, each analysis of another device region is another important tool for a future analysis of a complete device. One device region was analyzed here. For a simple diode during the recovery stage, other "circuit elements" include contact potentials at electrodes and an expanding DR characterized by a moving DRB. The expanding DR is not quantitatively analyzed in this paper, so the present work is only a tool for future work. However, the expanding DR is qualitatively discussed in Sections VIII and IX.

VII. Some Applications of the Modified Ohm's Law

The first application of the modified Ohm's law answers the question of whether a track is conductive. Note that the classical conductivity σ relates the current to the modified potential ψ (a somewhat abstract quantity), while the type of conductivity relevant to the present question is an effective conductivity relating the current to the true electrostatic potential φ. The two types of conductivities can be compared by numerically comparing the two types of potentials. The difference between these potentials is the logarithmic term in (14), so we will estimate this term for conditions under which this term is largest.

Consider a very heavy ion having an LET on the order of 40 MeV-cm^2/mg. The LET determines the linear track density (charge per unit length), but the volume density P depends on how concentrated (or narrow) the track is assumed to be. Note that after a short time (compared to the time over which significant charge collection occurs) of radial ambipolar diffusion, the distinction between an initially narrow track and an initially very narrow track diminishes in the absence of Auger recombination. The radial profile approaches a gaussian function, even if the initial profile is better described by some other function. Auger recombination can only further reduce the density, so it should be sufficiently conservative to neglect Auger recombination, assume a gaussian radial profile with a 0.1-μm characteristic radius, and then add a little margin. For such a profile and an LET of 40, the maximum (i.e., on the track center line) excess carrier density is slightly less than 10^{20}/cm^3. We will add some margin and call it 10^{22}/cm^3. Assuming that the doping density is at least 10^{14}/cm^3 (it is usually larger), the largest value of P throughout the quasi-neutral region exceeds the doping density by not more than eight orders of magnitude. Note that (10) gives a smaller A for a p-type region than for an n-type region, so we will assume a p-type region to estimate a larger value for the logarithmic term. The ratio of the electron-to-hole mobilities is typically between one and three. We will assume a ratio of three because this produces a larger value for the logarithmic term. Using these conservative estimates, the difference between the extreme values of the logarithmic term (the largest value in the quasi-neutral region minus the smallest value) is about one-fourth of a volt. Less conservative assumptions produce a smaller estimate. The difference between the extreme values of φ during a typical funneling process when the carrier density is very large is several volts. Therefore, while not highly accurate, we do obtain a rough approximation by replacing $\Delta\psi$ with $\Delta\varphi$ in the integrated equation $\Delta\psi = IR$, i.e., the effective conductivity can be approximated by the classical conductivity, at least for the purpose of calculating the total resistance between boundaries.

In addition to comparing the extreme variations of the two potentials, we can also compare more local variations relevant to the point form (13). We might speculate that variations in the logarithmic term at different locations could be important in regions where the variation in φ is small (i.e., in regions where the electric field is weakest). However, such regions are also characterized by small variations in $P+A$ (small compared to the eight orders of magnitude considered previously), so the variation in φ is still roughly the same as the variation in ψ. In fact, the longitudinal variation in ψ is precisely equal to the longitudinal variation in φ in regions where the track is longitudinally uniform.

It is concluded from the above discussion that, when investigating charge collection from ion tracks during times when funneling is strong (i.e., when there is a large voltage across the quasi-neutral region), the distinction between φ and ψ is small enough so that it can be ignored for the purpose of roughly estimating the conductivity. Therefore, the effective conductivity relating the current to φ is approximately the classical conductivity σ. Note that any effects on mobility, such as carrier–carrier scattering, also affect σ, and such nonlinear effects will prevent σ from being proportional to the carrier density. However, with the exception of those cases (if such cases exist) in which nonlinear effects are strong enough for σ to decrease with increasing carrier density, the denser track regions are the most conductive. It is clear that conduction is not confined exclusively to the outer track regions where the carrier density is less than the doping density.

It now becomes easy to see why equipotential surfaces in the presence of ion tracks during strong funneling are shaped like funnels. The conductivity function can be described as a highly conductive track above a low-conductivity region below the track. The track conductivity is large near the track center line, but smaller at larger radial distances. The electric field in the upper region is therefore weakest near the center line and stronger at larger radial distances. This implies that equipotential surfaces are further apart near the center line than at larger radial distances, hence the funnel shapes. The funnel shape is just Ohm's law.

Some other applications of the modified Ohm's law are incidental (and the conclusions are not very original), but are presented as a matter of curiosity because they are so simple. All discussions below tacitly assume that transients are slow enough so that we do not have to distinguish between I and I' in Fig. 7, i.e., capacitance effects are not important.

Note from (15) that the current is zero if and only if the left side is zero. Therefore, for the most general possible case represented in Fig. 7(a), open circuit conditions imply a ΔV equal to the logarithmic term. For the purpose of comparison with previously known results, we consider the special case of low-density conditions ($P_1, P_2 \ll$ doping density). Expanding the logarithmic term to first order in P_1/A and in P_2/A gives

$$\Delta V = \frac{B}{A} V_T \Delta P = \frac{\mu_e - \mu_h}{\mu_e n_0 + \mu_h p_0} V_T \Delta P.$$

This result is not new [8], but note how easily it can be derived from the general result (15).

For a final application, consider the device in Fig. 7(c) in which both active boundaries are ideal ohmic contacts. Contact potentials at the two contacts subtract out, so $\Delta V' = \Delta V$. Also, $P = 0$ at both boundaries, so (15) becomes $\Delta V' = IR$. We conclude that two ideal ohmic contacts create a passive device even if an ion track liberates carriers within. The open circuit voltage is zero and the short circuit current is zero. The device is just a resistor, but the resistance is variable when carriers are generated within the device. Incidentally, this example is another simple illustration of a current without a charge separation.

VIII. A Future Application

An anticipated future application of (and the primary motivation for) the present work is to improve the accuracy of an earlier, simple but crude, analysis of charge collection in diodes. The analysis, which is limited to high-density conditions (the carrier density greatly exceeds the doping density), calculates the gradient of the carrier density function at the DRB by solving the ambipolar diffusion equation, and this gradient is then used to calculate the minority carrier diffusion current. The analysis neglects DRB motion and equates total current to twice the minority carrier diffusion current. The latter statement is based on the observation that if the DR is reversed biased, then, to the extent that the DRB can be approximated as stationary, it blocks the majority carrier current. This means that the majority carrier drift and diffusion currents have the same absolute value at the DRB. Under high-density conditions, the electron and hole densities have nearly equal values and gradients at the DRB, so electron and hole drift currents are in the ratio of the mobilities and electron and hole diffusion currents are in the ratio of the mobilities. Therefore, majority carrier drift equal to majority carrier diffusion at the DRB implies that minority carrier drift equals minority carrier diffusion at the DRB. However, the two minority carrier currents add to instead of subtract from each other, so the total current at the DRB is twice the minority carrier diffusion current.

Limitations are discussed below, but it is interesting that, to the extent that the above approximations do apply, we can think of current as the cause and the voltage across the quasi-neutral region (or funneling) an effect. This terminology reflects the fact that the current can be roughly estimated from an independent analysis first, without prior knowledge of $\Delta\psi$, and this estimate can then be substituted into $\Delta\psi = IR$ to obtain an estimate of $\Delta\psi/R$. Physically, the DR responds to the current, so its voltage and the quasi-neutral region voltage respond to the current.

These simple concepts are sometimes useful for a low-order approximation [5], but this approximation is not accurate enough to properly distinguish between some cases that are distinguishable in computer simulation results. Examples of such cases (the effect of track length and doping type) are discussed later in this section, and some other examples (the effect of changing ion LET, etc.) are mentioned in the next section. A higher order approximation able to properly distinguish between such cases must recognize a mutual interaction between voltages and currents. The interaction not included in the low-order approximation is through DRB motion. The DRB moves as the DR expands during recovery, sometimes very fast as discussed in the next paragraph. This motion affects both minority-carrier and majority-carrier currents. From the point of view of minority carriers, a moving DRB resembles a moving vacuum cleaner which collects carriers faster than a stationary vacuum cleaner. From the point of view of majority carriers, the moving DRB resembles a barrier that pushes them along in front of it as it moves, creating a majority-carrier current. The DRB motion affects the current but is itself affected by the voltage distribution; hence there are mutual interactions that must be included to obtain a higher order approximation.

Influences on and by DRB motion are most striking when there is a fast partial DR recovery. An example of a fast partial recovery is seen, from computer simulation results, when the track is long enough to reach the lower electrode. A simulation treated a simple n^+–p device illustrated in Fig. 1, containing a shallow n^+ diffusion above a 100-μm-thick p-type substrate. The track was from a normal incident center hit and reached the lower electrode. The lower track end quickly clears away, producing a low-conductivity region below the track, but this region is narrow at early times after the track formation, so R is small. The simulation shows a greatly collapsed DR immediately after the hit, but a very fast partial recovery occurs so that the DR quickly takes up most of the device voltage in response to the small value of R. Prior to this partial recovery, the current is considerably larger than for a shorter (35 μm) track having the same LET, although only for the very short time required for the partial recovery. That there should be a large current prior to the partial recovery can be seen from either of two points of view. The first point of view recognizes that R is small and $\Delta\psi$ is large prior to the partial recovery. The second point of view recognizes that the DRB is moving fast prior to the recovery (the low-order approximation, which neglects DRB motion, cannot distinguish between a long track and a shorter track at such early times). Each point of view is valid, but neither point of view, taken by itself without the other, provides enough information to completely solve for all quantities.

A numerical comparison between the two cases is shown in Fig. 8, which plots accumulated collected charge $Q(t)$ as a function of time t for each of the two track lengths. The 35-μm case is the same as the n^+–p case discussed in Section IV, and the 100-μm case represents identical conditions (including LET) except for track length. The partial recovery for the long track occurs during the first few tenths of a nanosecond and accounts for the difference between the two curves at the end of this time. After this time (but before 10 ns), the currents are nearly equal for the two cases, so the difference between the two curves remains roughly constant in t. This roughly constant difference is the extra charge that the long-track case

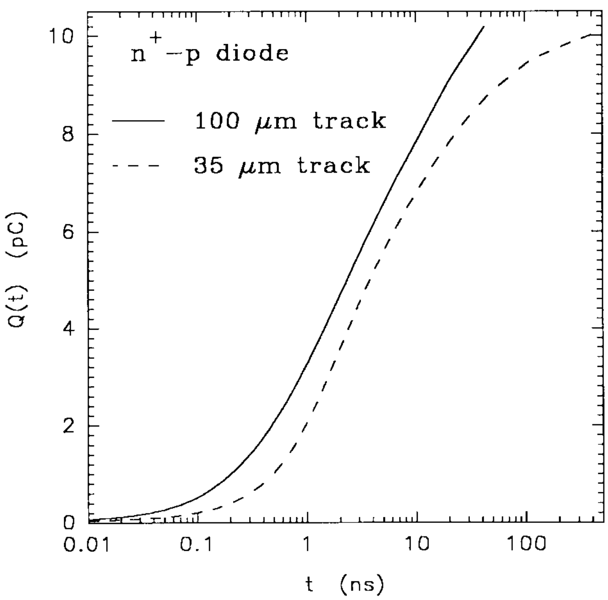

Fig. 8. Collected charge versus time produced by two track lengths in an n^+–p diode.

collected during the fast and brief partial recovery. At very late times (after about 10 ns), the difference between the two curves increases again, for an obvious reason having nothing to do with DRB motion; some carriers initially liberated on the more remote sections of the long track have had enough time to reach the DRB. The low-order approximation can predict this, but it cannot predict the noticeable difference between the two curves produced by the partial DR recovery at the earlier times.

Another example of a fast partial DR recovery is seen by going back to the 35-μm track, but interchanging p and n types in the diode. This is the same as the p^+–n case described earlier in Section IV. The DR width at 0.6 ns is about 1 μm for the p^+–n case, compared to about 0.1 μm for the n^+–p case, so DRB motion at early times is much faster for the former case. Because of this fast DRB motion, we should not expect the low-order approximation to be adequate for the p^+–n case during the earliest times. However, this approximation may (or may not, this remains to be seen) be roughly correct at later times after the initial partial recovery. It is therefore interesting to determine what the low-order approximation predicts with regards to how the two cases should compare. Simulations used the same electron mobility for the two cases and the same hole mobility for the two cases. Therefore, the low-order approximation, using the same input data as the simulations, uses the same ambipolar diffusion coefficient for both diode types. The low-order approximation does not make any other distinctions between the two cases when calculating the gradient of the carrier density function, so the same gradient is calculated for both cases. However, the current is calculated by multiplying the carrier density gradient by $2qD_h$ for the p^+–n case and by $2qD_e$ for the n^+–p case. The diffusion coefficients used in the simulations are in the ratio of $D_e/D_h \approx 2.6$, so the low-order approximation predicts the currents for the two cases to be in the ratio of about 2.6, with the n^+–p case producing the larger current.

Before comparing the above prediction to simulation results, we make another prediction from $\Delta\psi = IR$. Although expected to be more valid than the low-order approximation, this equation (taken by itself without any other analysis) suffers from a lack of input information; we do not know $\Delta\psi$ most of the time. However, this equation can make a prediction at sufficiently early times. Simulations show that, like the n^+–p case, the p^+–n case is also characterized by nearly all device voltage being across the quasi-neutral region at the start of the recovery stage, before the fast partial recovery. (Note that this applies to bulk diodes considered here but does *not* apply to epi diodes considered in an earlier paper [5] because another device region supports much of the voltage for the epi case.) Using $\Delta\psi \approx \Delta V$, we conclude that $\Delta\psi$ is about the same for the two cases at sufficiently early times, so the ratio of the currents for the two cases is the ratio of R for the two cases. A rough estimate for this ratio is obtained by imagining the ion track to be a short surrounded by a spreading resistance. A better estimate would include track resistance (the same for the two cases) and produce a ratio slightly closer to one than we will calculate here, but a rough estimate equates the ratio of the currents to the ratio of the spreading resistances described by the equilibrium carrier densities. The ratio of these resistances is the ratio of electron mobility to hole mobility. Therefore, we expect the ratio of the currents for the two cases to be roughly 2.6. This is the same ratio calculated in the previous paragraph, except that the two cases are reversed, i.e., the p^+–n case is predicted to produce the larger current.

The predictions for the ratio of the current for the n^+–p case to the current for the p^+–n case are summarized as follows. The low-order approximation, which is expected to be wrong at early times (because of a fast DRB motion) but is credible at later times, predicts a ratio of about 2.6. The modified Ohm's law predicts a ratio of roughly 1/2.6 at sufficiently early times but is inconclusive (because we do not know $\Delta\psi$) at later times. The currents calculated by computer simulations are compared in Fig. 9, which shows that the ratio roughly agrees with the 1/2.6 at early times and roughly agrees with the 2.6 at later times.

A future analysis able to account for influences on and by DRB motion requires enough equations to predict how the device voltage is divided between the DR and quasi-neutral region. This includes equations describing the DR, such as [7, (A9)], and equations describing the quasi-neutral region, such as (15) provided in the present paper.

IX. A Suggested Alternative to Funnels

The intention of this section is to discourage the practice of describing charge collection in terms of funnels and to suggest an alternate description. A distinction is made here between funneling (a process), a funnel length (a number), and a funnel (an object). The process is well established, and the historical name seems appropriate because funneling produces funnel-shaped equipotential surfaces. The number can be defined in terms of collected charge, although it is not essential that we do so, because collected charge can also be discussed using more

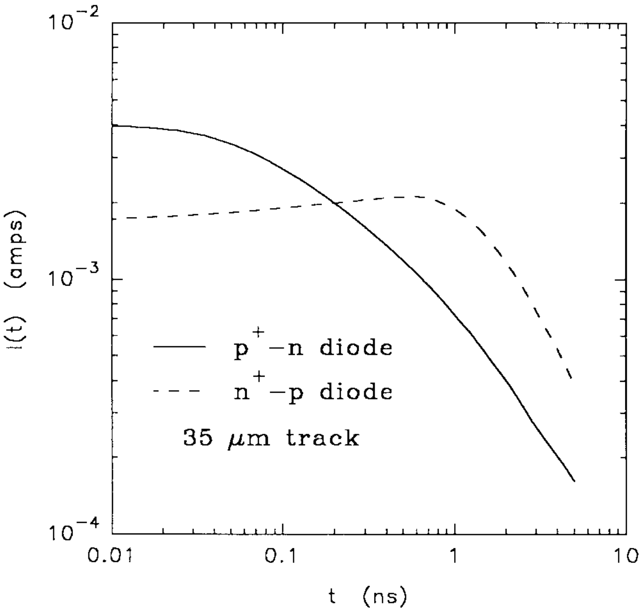

Fig. 9. Current versus time produced by a 35-μm track in two types of diodes.

basic terminology, e.g., in terms of collected charge normalized in any convenient way. The object is most poorly defined. Unlike DRB's, which can be identified in computer simulation results at each point in time during the recovery stage, funnels cannot. At any given time, each point in the device lies on some equipotential surface. We may see a few such surfaces or a lot in a figure, depending on the resolution, but every point in the device is on one. Which one of these funnel-shaped equipotential surfaces is the funnel? Fig. 4 shows an indication of a DRB and a lower track end, but no indication of the boundary of a funnel region. Some investigators describe a funnel as a strong-field drift region that promptly collects all charge contained within. However, the electric field is actually weak along a high-density track. Occasionally, we find in the literature a figure showing equipotential surfaces calculated by simulations, with one such surface identified as the funnel. However, the identification is arbitrary and an inspection of the spacing between equipotential surfaces shows that regions identified as funnels in the literature are actually regions where the electric field is weakest (the electric field is weakest where equipotential surfaces are farthest apart). Instead of a strong-field drift region, a better description is a weak-field ambipolar region. Drift currents appear in low-conductivity regions via strong electric fields (modulated by spreading effects) and in high-conductivity regions via weak electric fields; they are not confined to regions called funnels. A funnel can only serve as some kind of symbolic description. A more literal description would lead to a better understanding of charge collection if it is simple enough to be understood, at least qualitatively. Although quantitatively difficult, a qualitatively simple literal description is proposed below.

For a simple but literal qualitative description, we first consider initial conditions. Details of the processes occurring during the collapse stage may not yet be well understood; it is not even clear yet whether there is a DRB during the initial charge rearrangement. However, by 10 ps we are already sufficiently far into the recovery stage so that analytical calculations of collected charge, which assume conditions describing the recovery stage, agree with simulation results [5]. Furthermore, computer calculations of collected charge show only a small amount of charge collected during this time [5], [9]. Therefore, *if* our interest is in collected charge accumulated up to a given time (as opposed to some other possibly interesting quantities, such as peak current or current rise time), and *if* our accuracy requirements are satisfied when the error in this charge is a small fraction of the total amount of charge that will eventually be collected, then we can assume that the recovery stage applies to the entire charge-collection process. During this stage, there is a DRB as illustrated in Fig. 2.

Charge collection during the recovery stage can be described by starting with a low-order approximation and then describing corrections that must be made to obtain a higher order approximation. The low-order approximation neglects DRB motion. During high-density conditions (the carrier density greatly exceeds the doping density at the locations on the DRB where most of the current flows), the total current is twice the minority carrier diffusion current. The factor of two includes the drift current as discussed in the previous section. At later times, after the track dissipates to the extent that high-density conditions are replaced by the opposite extreme (low-density conditions), the total current is just the minority carrier diffusion current.

A correction needed to improve accuracy is to include the effects of DRB motion. This correction explains the influence that various parameters (e.g., ion LET, doping density, etc.) have on the normalized (to ion LET or linear track density) collected charge or current. The influence of doping density is particularly predictable because there are no competing effects; all effects are in the same direction. Increasing the doping density decreases mobility and lifetime. These two effects reduce the minority carrier diffusion current, and the low-order approximation can account for this. However, increasing the doping density also tends to reduce the current for another reason, which the low-order approximation cannot account for. This is due to a decrease in DRB motion [5], because the final width that the DR approaches during recovery is less. The influence of ion LET is slightly more complicated, because there may be competing effects. If the LET is reduced, but still large enough for high-density conditions to persist for an extended time, the first (earliest in time) effect seen is an increased (compared to a higher LET case) normalized current, because of a faster moving DRB [5]. The DRB moves faster because the DR can recover faster from the lower LET ion. A competing effect, which is important at a later time, is that high-density conditions have a shorter duration. Depending on the individual case, late times might show the normalized collected charge (as well as the normalized current) to be smaller for the lower-LET case. Changing doping types can also introduce competing effects for bulk diodes. Comparing the p^+-n case to the n^+-p case, we find that the former case may have an early tendency to produce a larger current because of a fast moving DRB, but a later tendency to produce a smaller current because of a smaller minority carrier diffusion coefficient (as discussed in the previous section). Increasing the bias voltage tends to increase the current at early times, because the DRB moves faster. Increasing the track length has one obvious effect at late times, which can be predicted by the low-order approximation, but it also has a less obvious effect at early times, which cannot be predicted by the low-order approximation. The long track produces a larger current at early times due to a faster DR recovery, as discussed in the previous section.

Additional work is needed to convert the above qualitative discussion into a quantitative model, but the qualitative discussion may add some physical insight into the problem of charge collection in diodes.

X. Summary

This work is intended to assist future theoretical modeling efforts by providing one quantitative relationship which is necessary (though, not sufficient) to determine how a device voltage is divided between distinct regions. This is (15), which relates the carrier density and potential to the current through the quasi-neutral region. This equation includes all currents: electron and hole, drift, and diffusion. While this one relationship is not enough for a quantitative analysis of a complete device, a fairly complete qualitative discussion was given for charge collection in diodes.

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