Assessing Part-to-Part Variation for Destructive Single-Event Effects

Ray Ladbury
GSFC Radiation Effects and Analysis Group

Presented by Ray Ladbury at the 2012 SEE Symposium, April 3-5, 2012, LaJolla, CA
Why Worry about Variable SEE Failure Rates?

• Goal of destructive Single-Event Effects (SEE) Radiation Hardness Assurance (RHA) is avoiding risk
• Sometimes risk cannot be avoided
  – Heritage hardware may have used obsolete test or qualification methods
    • Examples: Single-event gate rupture (SEGR) dependence on ion range, Z, etc.; Single-event Latchup (SEL) susceptibility at cryo...
    • Does the risk warrant a redesign
    • What about risks to missions already using the hardware.
  – Continual pressure from designers to assume more risk
    • “Please, please, pretty please! Can’t you run that MOSFET with ±10 V on the gate?
• Problems with risk estimation for destructive SEE
  – Destructive SEE mechanism models still evolving—so rate estimation is crude
  – Every data point represents destruction of an expensive component
    • Makes testing expensive
    • Accumulating statistics must be accumulated across many devices
      – How do we estimate effects of part-to-part variation on test measurements
      – How do we carry out RHA when part-to-part variation it may be dwarfed by Poisson fluctuations on fluence to failure.

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Outline of Talk

• I. Statistics of Destructive SEE Fluences
• II. Models Considered
• III. Results
• IV. RHA Implications
• V. Avenues for Progress
Statistics of Destructive SEE

• **DSEE are Poisson in particle fluence**
  – Implies fluence to first occurrence is distributed exponentially with the same mean as the Poisson distribution
  – Can only occur once per part

• **Exponential Distribution Properties**
  – If mean=\( \mu \), standard deviation, \( \sigma = \mu \)
  – Skew=2
  – Excess kurtosis=6
  – Implications for SEE testing
    • Fluences for any single part may deviate significantly from the mean even if all parts identical
    • How do we ascertain mean and part-to-part variation when fluences to failure are so broadly distributed

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Models of Variability

Model 1: Lognormal variation

- Let expected fluence to failure vary
  - $\langle F \rangle \sim LNORM(F, m, s)$
  - How large must $s$ be before variability detectable?
  - Can we infer or bound $s$?

- Expected fluence is bimodal
  - $\langle F \rangle \sim 0.5[LNORM_1(F, m_1, s_1) + LNORM_2(F, m_2, s_2)]$
  - Is bimodality detectable?
  - How much must peaks differ to detect second mode?
  - Can we distinguish bimodality from model 1?

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Method

Monte Carlo Study

- Explore problem’s phase space by randomly sampling points
- For n events, errors scale as $n^{-1/2}$
- Generate 10000 events (1% errors) and examine convergence of distribution moments with sample size m
  - Small sample size (m<30) is relevant for SEE testing
  - Large sample size (m>30) relevant for examining bias in sample estimators
- Introduce increasing variability and count events detected
  - Efficiency=% events detected
  - False triggers—analysis indicates part-to-part variation where there is none

Hyperexponential and moments

- Hyperexponential Distribution
  - Mixture of exponential distributions with different means
  - Also called phase distributions
  - If phases close, looks exponential
- Method of moments
  - Nth moment of distribution $p(x)$
    - $N$th Moment = $\int x^N p(x) dx$
  - 1st moment= mean
  - 2nd centered moment=variance=$\sigma^2$
  - Skew—related to 3rd centered moment
  - Kurtosis—related to 4th centered moment
  - Skew and kurtosis normalized to $\sigma$
- Look at convergence of moments with m
  - Mean, $\mu$
  - $\sigma/\mu$~1 for exponential
  - Skew, Kurtosos (usually require large m)

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Skew and Kurtosis?

• First 2 moments are familiar
  – Mean is a location parameter
  – Variance/ $\sigma$ measure width
• What about skew?
  – Negative skew—left tail thicker; mode to the right of mean
  – Positive skew—right tail thicker; mode to left of mean
• Kurtosis—this one gives folks trouble
  – Measures relative amount of probability in peak and tails of distribution
  – Convention: Normal has zero kurtosis
  – Kurtosis$>$0: Thicker tails than Normal
  – Kurtosis$<$0: Thinner tails than Normal
• Sample skew and kurtosis are biased estimators of population values
  – Excel formulas are bias-corrected

• Weibull forms illustrate higher moments
  – $s<1$: $\sigma/\mu >1$, large skew, kurtosis
  – $s=1$: exponential-- $\sigma/\mu =1$, skew=2, kurtosis=6
  – $s=2.252$: $\sigma/\mu =0.47$, large skew, kurtosis=0
  – $s=3.6$: $\sigma/\mu >1$, skew=0, kurtosis=-0.28
  – $s>3.6$: $\sigma/\mu$ increasing, skew increasingly negative, kurtosis$>$0 again @$s=5.8

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Results: How Challenging is the Problem?

- Monte Carlo runs with 900000 events reproduces exponential trends well
  - Event counts at high failure fluence are lower, but errors ~10%

- Hyperexponential due to bimodal distribution distinguishable from exponential trend

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Results: How Challenging is the Problem?

- Hyperexponential due to lognormal part-to-part variation with lognormal std. deviation $sLN=0.2$ not distinguishable from exponential trend
  - Cannot detect differences in fluence to failure on order of 20%

- Hyperexponential due to lognormal part-to-part variation with lognormal std. deviation $sLN=0.5$ distinguishable from exponential trend
  - Differences are subtle
  - Can barely detect differences in fluence to failure on order of 53%

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Large Differences Are Detectable

- When sLN=1, part-to-part variation is comparable in scale to Poisson fluctuations in fluence to failure
  - Distribution of fluence to failure is easily distinguishable from exponential trend.

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Sample Mean a Good Estimator

- For exponential variation with constant mean
  - Sample mean underpredicts mean slightly for small samples, but agreement good

- For variable mean (sLN=0.5), convergence of sample mean good, but
  - Significant overestimates possible for small sample size

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Sample Standard Deviation Converges Slowly

- Exponential with constant mean
  - Sample Standard deviation underpredicts population std. dev.
  - Convergence is slower than for mean

- For variable mean (sLN=0.5), little difference from true exponential case, except for large values.
  - Significant overestimates possible for small sample size

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\( \sigma/\mu \) Distinguishes Exponential from Hyperexponential

- For Sample size 15, assume \( \sigma/\mu > 1.05 \) implies large part-to-part variation
  - Catches 50% of real cases
  - Unfortunately: 26% of true exponential cases cause false triggers

- For Sample size 15, assume \( \sigma/\mu > 1.11 \) implies large part-to-part variation
  - Catches 50% of real cases
  - Unfortunately: 16% of true exponential cases cause false triggers

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Sample Skew Less Effective Discriminator

- For Sample size 15, assume Skew>1.54 implies large part-to-part variation
  - Catches 50% of real cases
  - Unfortunately: 32% of true exponential cases cause false triggers

- For Sample size 15, assume σ/µ>1.9 implies large part-to-part variation
  - Catches 50% of real cases
  - Unfortunately: 26% of true exponential cases cause false triggers

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RHA Implications

• Small part-to-part variation will not be detectable for reasonable sample sizes due to Poisson fluctuations in fluence to failure.
• Mean fluence to failure converges nicely even for moderate sample size
• Standard Deviation converges more slowly
• Higher moments cannot be estimated reliably for sample sizes <30-50
• Detecting part-to-part variation efficiently results in false positives
  – Ratio $\sigma/\mu$ is most effective discriminator in study
  – Skew is less effective due to systematic underestimation for small sample sizes
  – Kurtosis is ineffective due to systematic underestimation for small sample sizes
• Serious distribution pathologies in part-to-part variation (bimodality) can be detected if severe enough
• Variability and pathologies can be bounded by Monte Carlo techniques.
Conclusions and Avenues for Progress

• Current RHA approaches to SEGR do a poor job of evaluating risk.
  – Usually overly conservative, but,
  – Poorly understood mechanisms sometimes result in mistakes
• Improved approach to risk estimation possible, but complicated
  – Part-to-part variation in destructive SEE hard to measure due to Poisson nature
    • Undetectable if not large
    • Likely results in false alarms
  – Other Questions
    • Is variability greatest near SEGR threshold?
    • Is variability different for different LETs? Different Ions? Different Range?
• Other Approaches
  – Bayesian treatment with prior modeled on pre-rad Breakdown VGS distribution
  – Physics-based modeling
  – All approaches suggest variation of Lethal Ion Approach (Titus-1999), (Lauenstein-2010)