Assessing Part-to-Part Variation for Destructive Single-Event Effects

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Why Worry about Variable SEE Failure Rates?

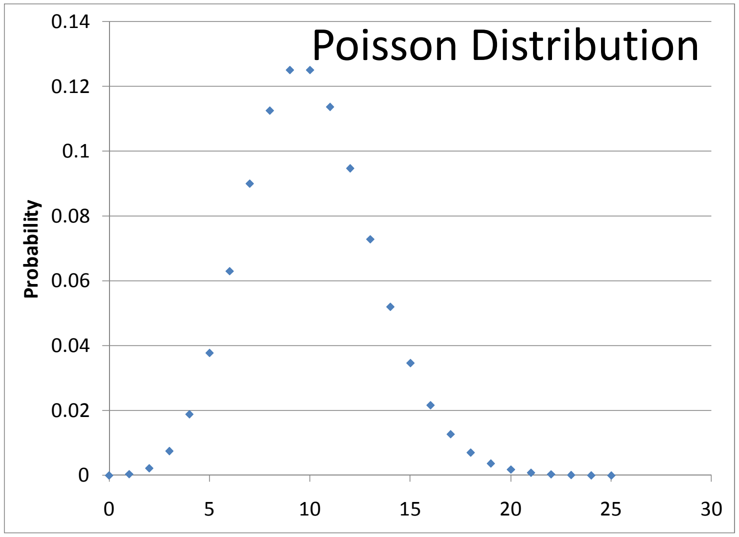
- Goal of destructive Single-Event Effects (SEE) Radiation Hardness Assurance (RHA) is avoiding risk
- Sometimes risk cannot be avoided
  - Heritage hardware may have used obsolete test or qualification methods
    - Examples: Single-event gate rupture (SEGR) dependence on ion range, Z, etc.; Single-event Latchup (SEL) susceptibility at cryo…
    - Does the risk warrant a redesign
    - What about risks to missions already using the hardware.
  - Continual pressure from designers to assume more risk
    - "Please, please, pretty please! Can't you run that MOSFET with ±10 V on the gate?
- Problems with risk estimation for destructive SEE
  - Destructive SEE mechanism models still evolving—so rate estimation is crude
  - Every data point represents destruction of an expensive component
    - Makes testing expensive
    - Accumulating statistics must be accumulated across many devices
      - How do we estimate effects of part-to-part variation on test measurements
      - How do we carry out RHA when part-to-part variation it may be dwarfed by Poisson fluctuations on fluence to failure.

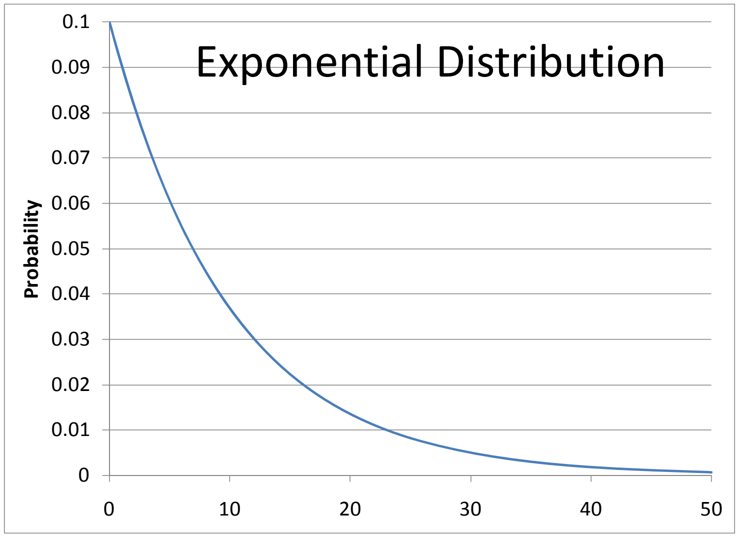
Outline of Talk

- I. Statistics of Destructive SEE Fluences
- II. Models Considered
- III. Results
- IV. RHA Implications
- V. Avenues for Progress

Statistics of Destructive SEE

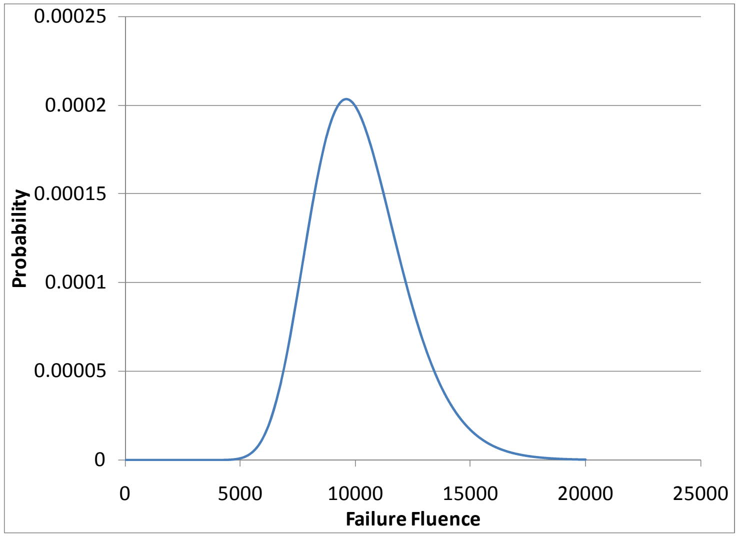
- DSEE are Poisson in particle fluence
  - Implies fluence to first occurrence is distributed exponentially with the same mean as the Poisson distribution
  - Can only occur once per part
- Exponential Distribution Properties
  - If mean=μ, standard deviation, σ=μ
  - Skew=2
  - Excess kurtosis=6
  - Implications for SEE testing
    - Fluences for any single part may deviate significantly from the mean even if all parts identical
    - How do we ascertain mean and part-to-part variation when fluences to failure are so broadly distributed





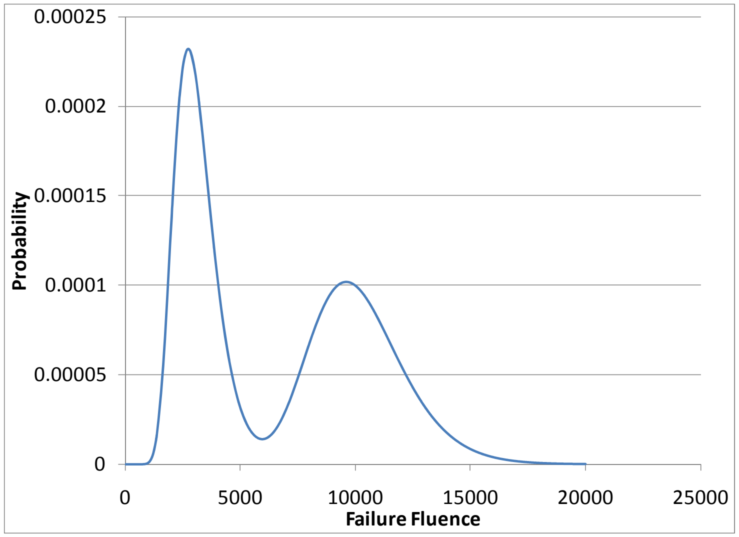
Models of Variability

Model 1: Lognormal variation



- Let expected fluence to failure vary
  - ⟨F⟩~LNORM(F,m,s)
  - How large must s be before variability detectable?
  - Can we infer or bound s?

Model 1: Lognormal variation



- Expected fluence is bimodal
  - ⟨F⟩~0.5*[$LNORM_1(F,m_1,s_1)$+$LNORM_2(F,m_2,s_2)$]
  - Is bimodality detectable?
  - How much must peaks differ to detect second mode?
  - Can we distinguish bimodality from model 1?

Method

Monte Carlo Study

- Explore problem's phase space by randomly sampling points
- For n events, errors scale as $n^{-1/2}$
- Generate 10000 events (1% errors) and examine convergence of distribution moments with sample size m
  - Small sample size (m<30) is relevant for SEE testing
  - Large sample size (m>30) relevant for examining bias in sample estimators
- Introduce increasing variability and count events detected
  - Efficiency=% events detected
  - False triggers—analysis indicates part-to-part variation where there is none

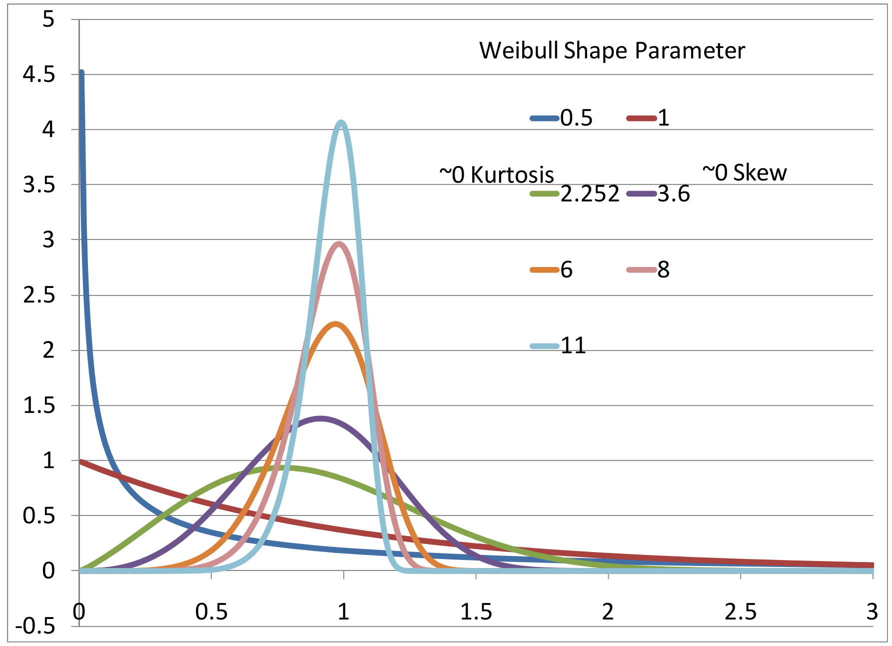
Hyperexponential and moments

- Hyperexponential Distribution
  - Mixture of exponential distributions with different means
  - Also called phase distributions
  - If phases close, looks exponential
- Method of moments
  - Nth moment of distribution *p(x)*

    $\text{Nth Moment} = \int x^N p(x)dx$
  - 1^{st} moment= mean
  - 2^{nd} centered moment=variance=σ^2
  - Skew—related to 3^{rd} centered moment
  - Kurtosis—related to 4^{th} centered moment
  - Skew and kurtosis normalized to σ
- Look at convergence of moments with m
  - Mean, μ
  - σ/μ~1 for exponential
  - Skew, Kurtosos (usually require large m)

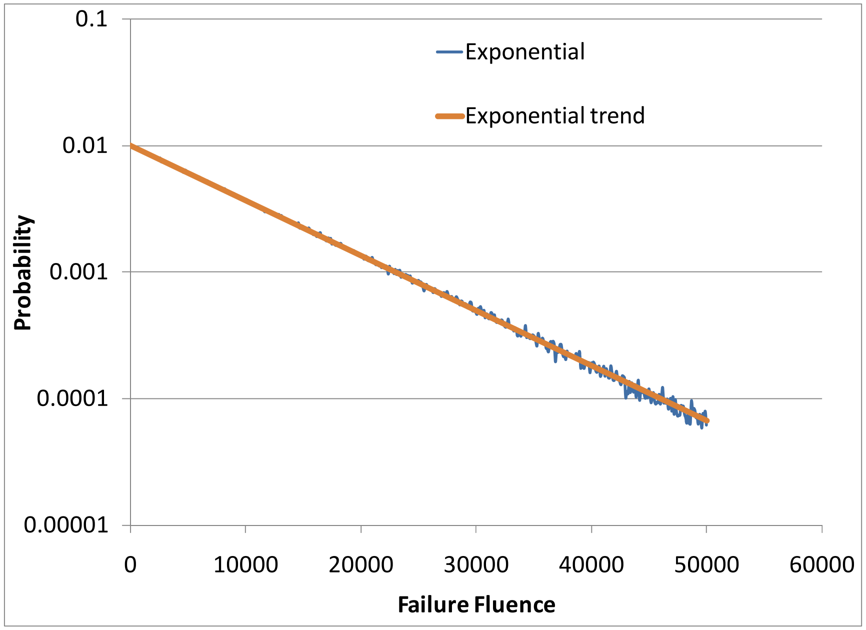
Skew and Kurtosis?

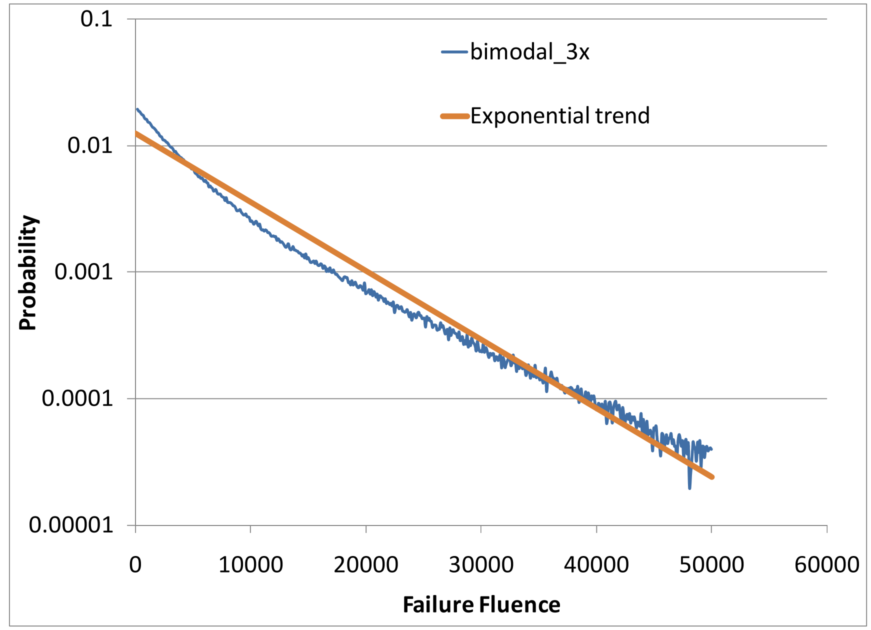
- First 2 moments are familiar
  - Mean is a location parameter
  - Variance/ σ measure width
- What about skew?
  - Negative skew—left tail thicker; mode to the right of mean
  - Positive skew—right tail thicker; mode to left of mean
- Kurtosis—this one gives folks trouble
  - Measures relative amount of probability in peak and tails of distribution
  - Convention: Normal has zero kurtosis
  - Kurtosis>0: Thicker tails than Normal
  - Kurtosis<0: Thinner tails than Normal
- Sample skew and kurtosis are biased estimators of population values
  - Excel formulas are bias-corrected



- Weibull forms illustrate higher moments
  - s<1: σ/μ >1, large skew, kurtosis
  - s=1: exponential-- σ/μ =1, skew=2, kurtosis=6
  - s=2.252: σ/μ =0.47, large skew, kurtosis=0
  - s=3.6: σ/μ >1, skew=0, kurtosis=-0.28
  - s>3.6: σ/μ increasing, skew increasingly negative, kurtosis>0 again @s=5.8

Results: How Challenging is the Problem?

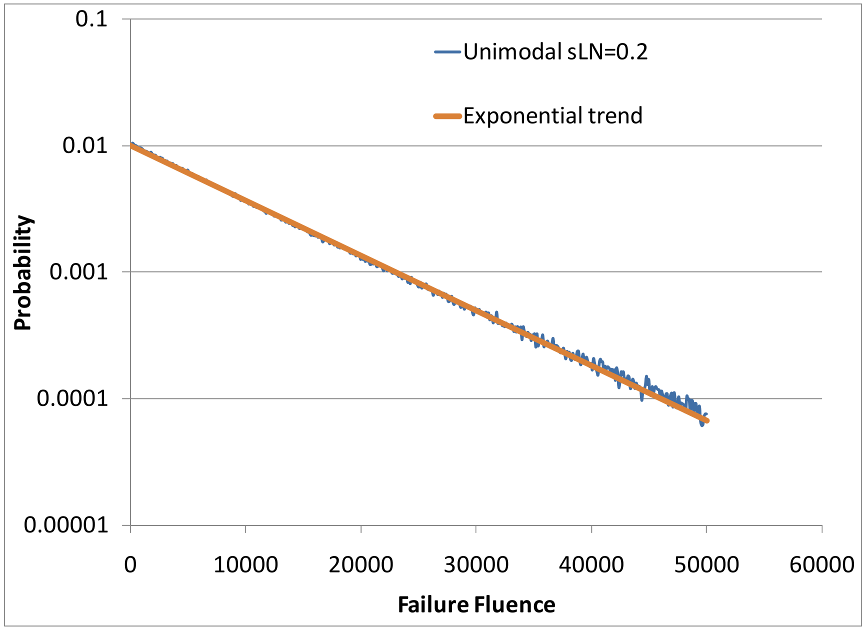




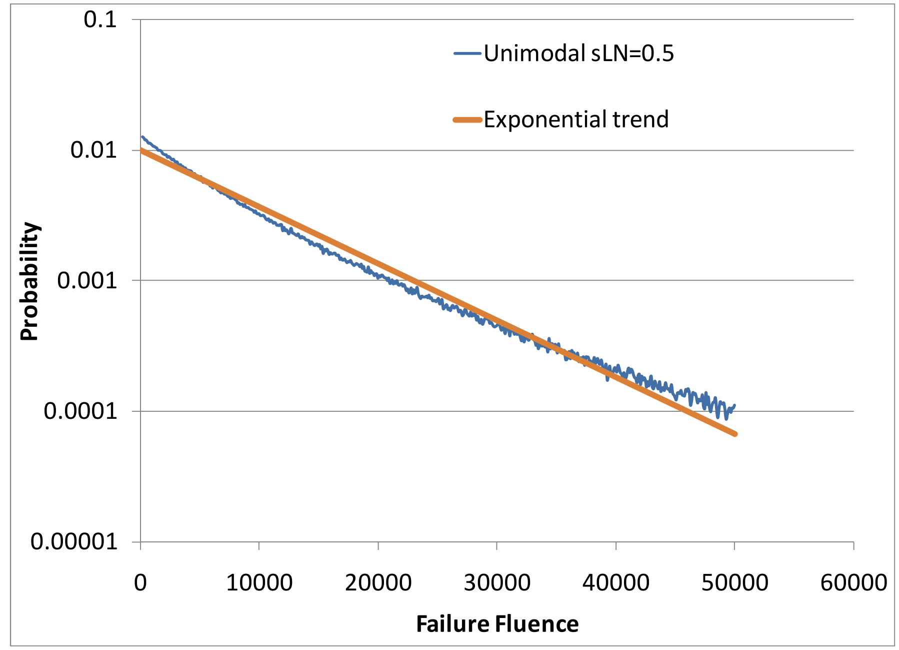
- Monte Carlo runs with 900000 events reproduces exponential trends well
  - Event counts at high failure fluence are lower, but errors ~10%

- Hyperexponential due to bimodal distribution distinguishable from exponential trend

Results: How Challenging is the Problem?

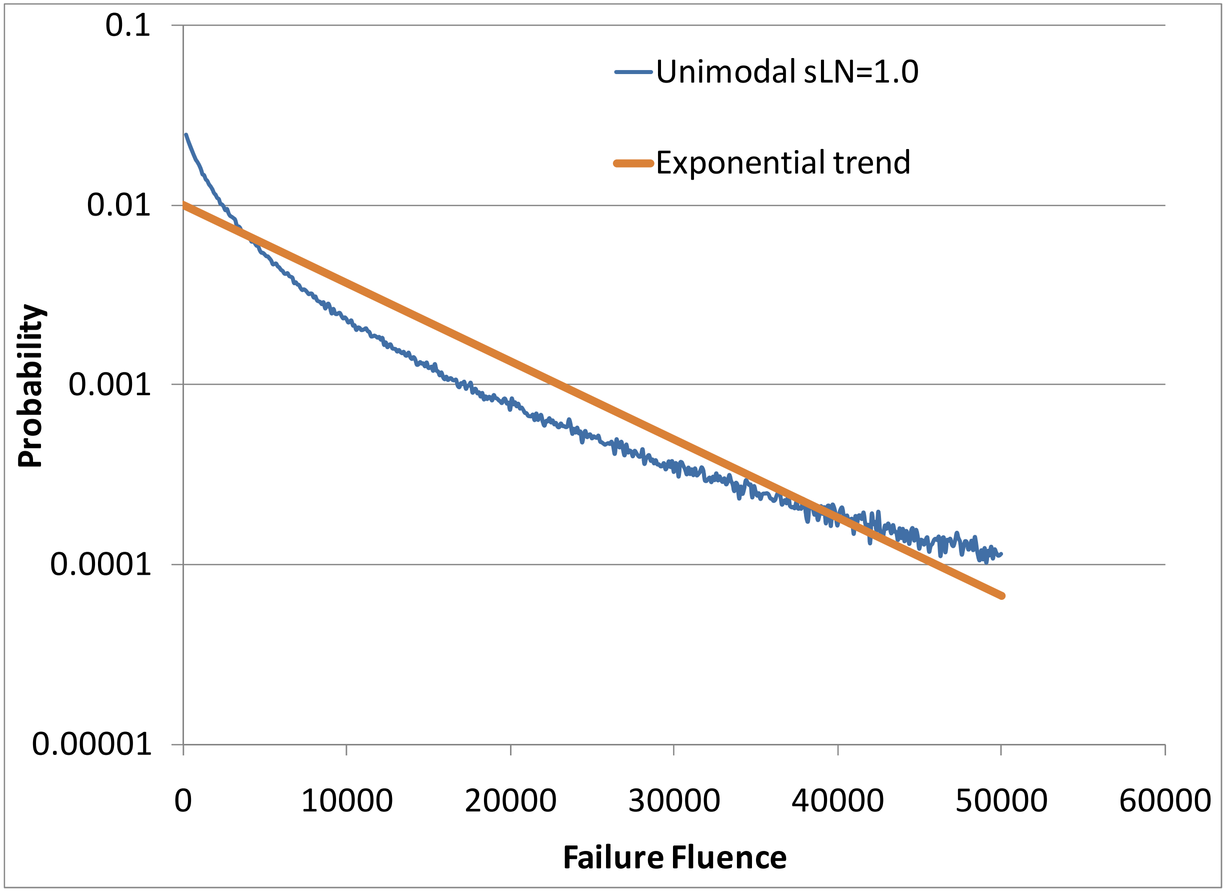


- Hyperexponential due to lognormal part-to-part variation with lognormal std. deviation sLN=0.2 not distinguishable from exponential trend
  - Cannot detect differences in fluence to failure on order of 20%



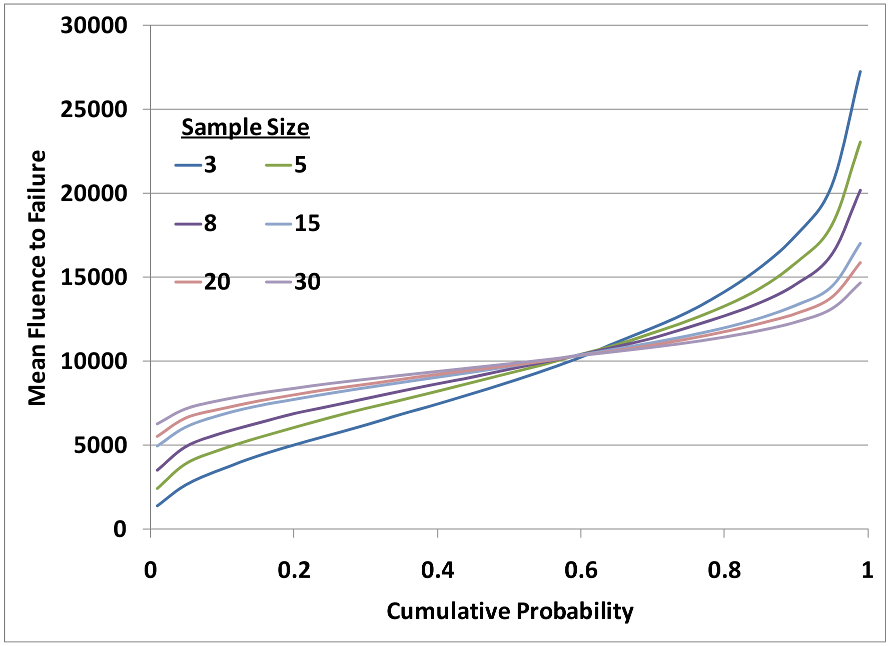
- Hyperexponential due to lognormal part-to-part variation with lognormal std. deviation sLN=0.5 distinguishable from exponential trend
  - Differences are subtle
  - Can barely detect differences in fluence to failure on order of 53%

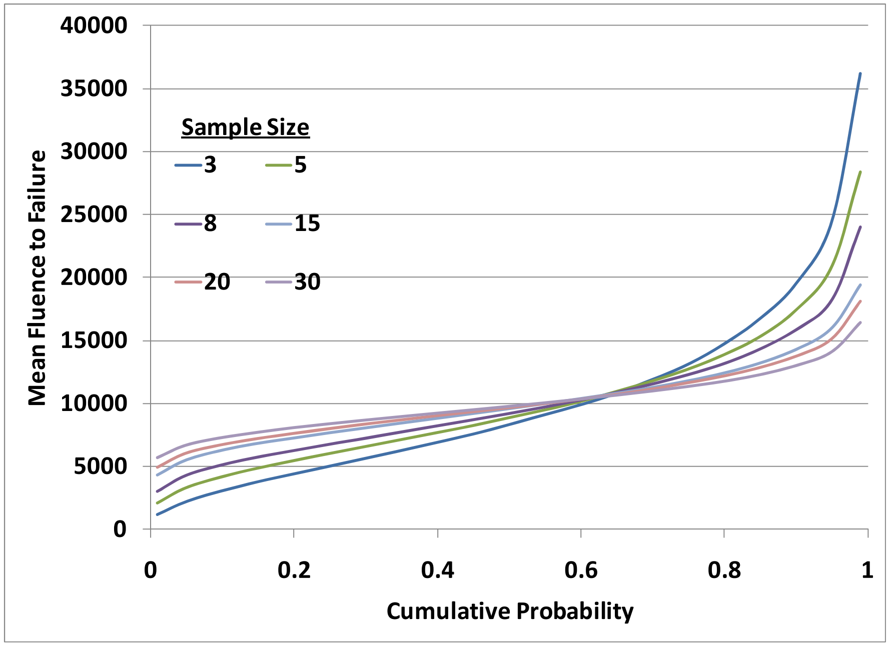
Large Differences Are Detectable



- When sLN=1, part-to-part variation is comparable in scale to Poisson fluctuations in fluence to failure
  - Distribution of fluence to failure is easily distinguishable from exponential trend.

Sample Mean a Good Estimator

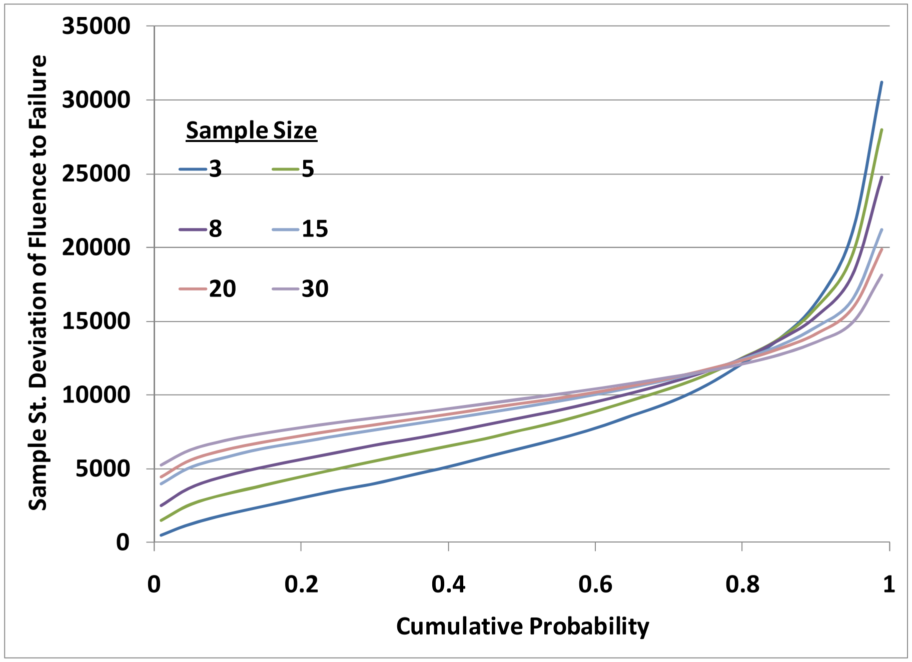


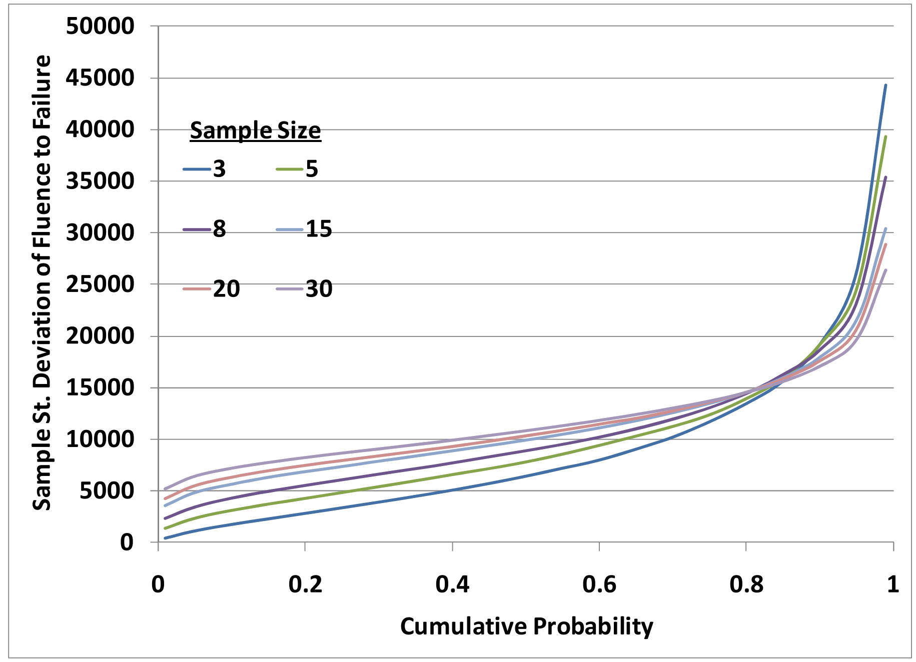


- For exponential variation with constant mean
  - Sample mean underpredicts mean slightly for small samples, but agreement good

- For variable mean (sLN=0.5), convergence of sample mean good, but
  - Significant overestimates possible for small sample size

Sample Standard Deviation Converges Slowly

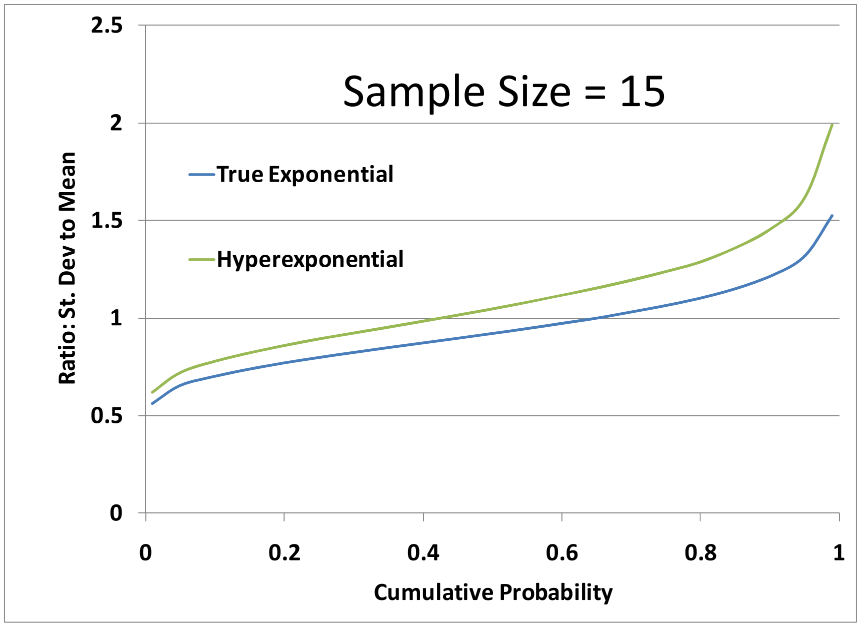


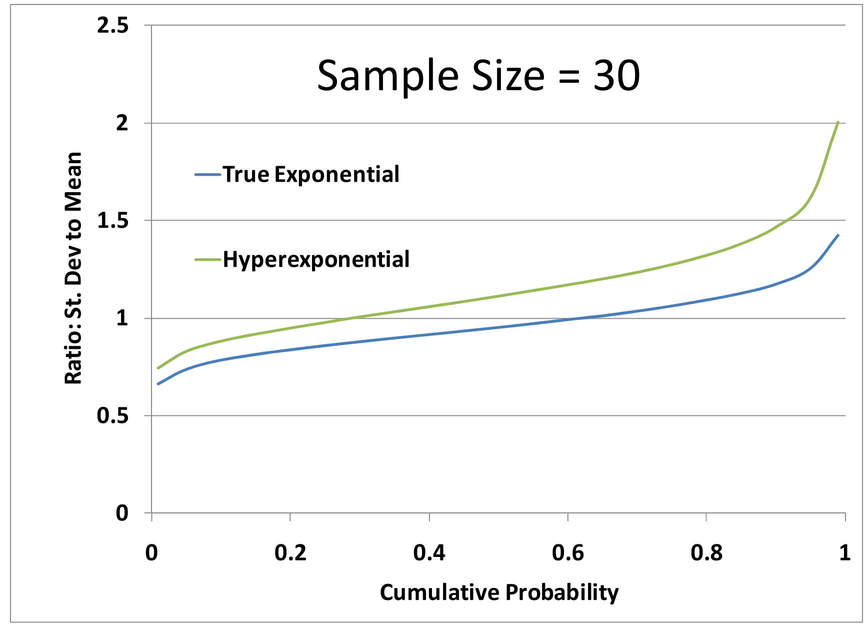


- Exponential with constant mean
  - Sample Standard deviation underpredicts population std. dev.
  - Convergence is slower than for mean

- For variable mean (sLN=0.5), little difference from true exponential case, except for large values.
  - Significant overestimates possible for small sample size

σ/μ Distinguishes Exponential from Hyperexponential

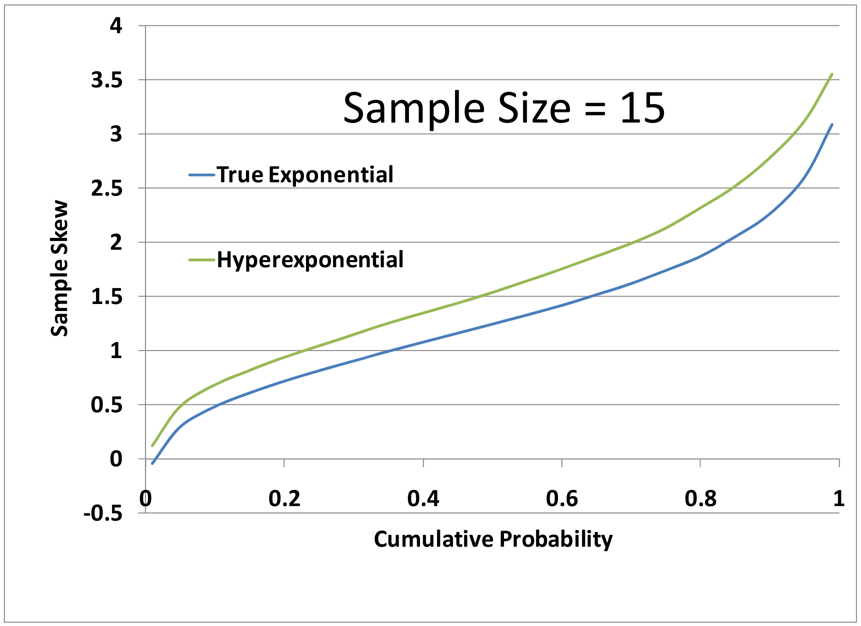


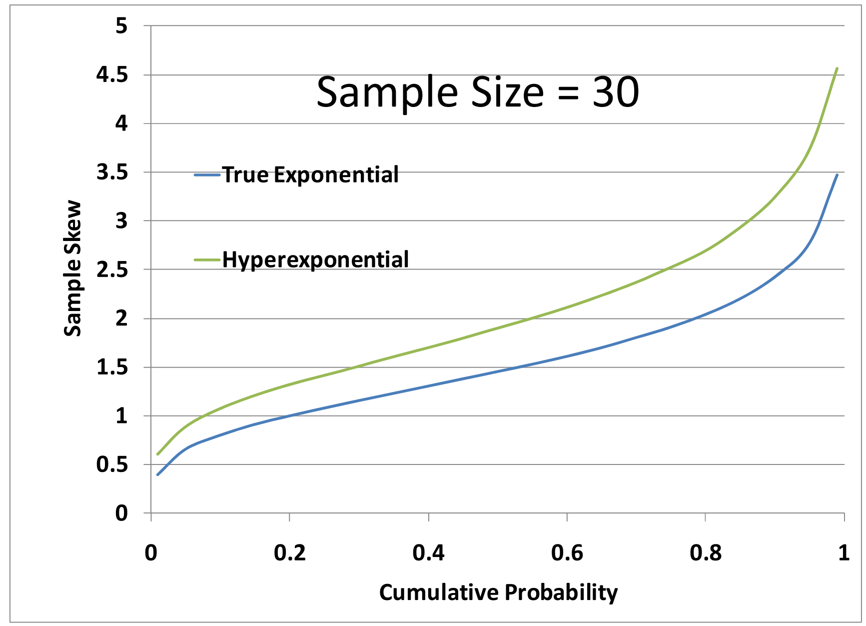


- For Sample size 15, assume σ/μ>1.05 implies large part-to-part variation
  - Catches 50% of real cases
  - Unfortunately: 26% of true exponential cases cause false triggers

- For Sample size 15, assume σ/μ>1.11 implies large part-to-part variation
  - Catches 50% of real cases
  - Unfortunately: 16% of true exponential cases cause false triggers

Sample Skew Less Effective Discriminator





- For Sample size 15, assume Skew>1.54 implies large part-to-part variation
  - Catches 50% of real cases
  - Unfortunately: 32% of true exponential cases cause false triggers

- For Sample size 15, assume $\sigma/\mu>1.9$ implies large part-to-part variation
  - Catches 50% of real cases
  - Unfortunately: 26% of true exponential cases cause false triggers

RHA Implications

- Small part-to-part variation will not be detectable for reasonable sample sizes due to Poisson fluctuations in fluence to failure.
- Mean fluence to failure converges nicely even for moderate sample size
- Standard Deviation converges more slowly
- Higher moments cannot be estimated reliably for sample sizes <30-50
- Detecting part-to-part variation efficiently results in false positives
  - Ratio σ/μ is most effective discriminator in study
  - Skew is less effective due to systematic underestimation for small sample sizes
  - Kurtosis is ineffective due to systematic underestimation for small sample sizes
- Serious distribution pathologies in part-to-part variation (bimodality) can be detected if severe enough
- Variability and pathologies can be bounded by Monte Carlo techniques.

Conclusions and Avenues forProgress

- Current RHA approaches to SEGR do a poor job of evaluating risk.
  - Usually overly conservative, but,
  - Poorly understood mechanisms sometimes result in mistakes
- Improved approach to risk estimation possible, but complicated
  - Part-to-part variation in destructive SEE hard to measure due to Poisson nature
    - Undetectable if not large
    - Likely results in false alarms
  - Other Questions
    - Is variability greatest near SEGR threshold?
    - Is variability different for different LETs? Different Ions? Different Range?
- Other Approaches
  - Bayesian treatment with prior modeled on pre-rad Breakdown VGS distribution
  - Physics-based modeling
  - All approaches suggest variation of Lethal Ion Approach (Titus-1999), (Lauenstein-2010)